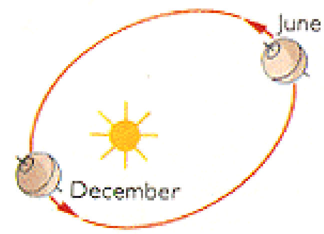
Lab 4: Modeling Temperature Data

In this lab, you'll model data with sine and cosine functions. It's worth 25 points.

Why would a periodic function like sine or cosine be particularly useful for modeling temperatures? For one, temperatures are cyclical. In the Northern Hemisphere, it tends to get cold in the winter and warmer in the summer. But why? In the southern hemisphere, our winter is their summer, even though the earth is closest to the sun in December.

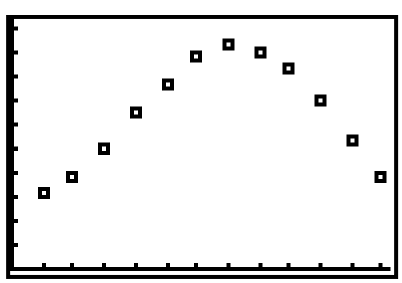
The reason: The Northern Hemisphere's summer is when the North Pole is tilted towards the sun[a], while the Southern Hemisphere has winter (and vice versa). Thus, even though the Northern Hemisphere is very far away from the sun during our summer, we're tilted toward the sun. Therefore, it's warmer in our half than in the Southern Hemisphere. The reverse is true in the winter—check out the diagram at right to help you visualize[b].

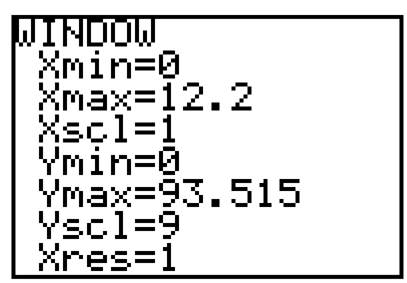


So, since temperatures are tied to the seasons, and seasons are tied to the orbit of the earth, and the orbit of the earth is elliptical...well, that makes it a prime candidate for a periodic function! Here is a listing of average monthly high temperatures for Chicago[c]:

Month	**Average High Temperature**
January	**29**
February	**34**
March	**46**
April	**59**
May	**70**
June	**80**
July	**84**
August	**82**
September	**75**
October	**63**
November	**48**
December	**34**

Let's start by constructing a scatter plot of these data in your TI. To make the data quantitative, let January be 1, February be 2, and so forth, until December's 12.[d] You can use the window given at the far right.

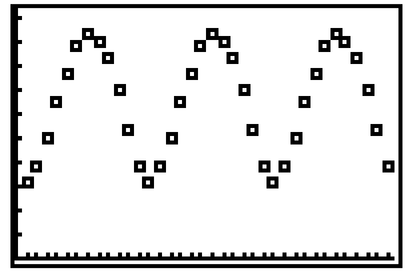




[a] The earth tilts at approximately 23.45°from vertical.
[b] That diagram is a *tad* misleading...the sun is actually closer to the center of our orbit that it appears there.
[c] Data from http://www-rohan.sdsu.edu/~jmahaffy/courses/f00/math122/lectures/trig/trig.html
[d] If you've never done a TI scatter plot before, consult your guide book. If you lost it, visit http://www.ticalc.org/basics/calculators/#3 to get the .pdf version. And let me know if you have any trouble!

I've graphed additional years here, and you can really see how the data looks sinusoidal (like a sine or cosine curve)[e].

So let's try to model this curve…



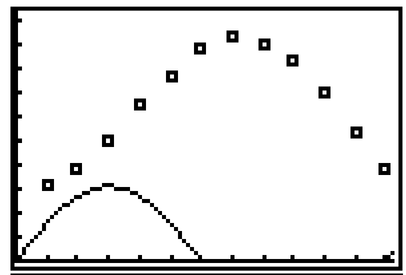
Task 1 (2 pts): To find the amplitude of this curve, find half the difference between the highest and lowest values. [Turn to the last page to write down your answer.]

Task 2 (2 pts): This curve repeats after 12 months, and therefore has a period of 12 months. Use that fact to find the value of ***k***. [Turn to the last page to write down your answer.]

Task 3 (no pts):

All right! Let's enter our curve $y = a\sin(kx)$ into Y1 and see what it looks like. Our window can stay the same, since the data already fits in it very well. Go ahead and try it!

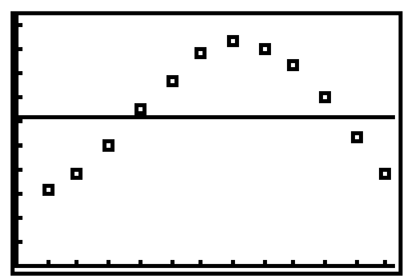
You can see that the curve actually looks right...we're just not looking at it in the right place! We need to tweak our curve a little bit, with a vertical translation and a phase shift.



Task 4 (2 pts): Remember that the sine and cosine curves are centered on the *x*-axis...that is, they oscillate at a fixed distance above (and below) $f(x) = 0$. This is because, with amplitude ***a***, the curve will be centered at the point directly between ***a*** and –***a***...which is 0.

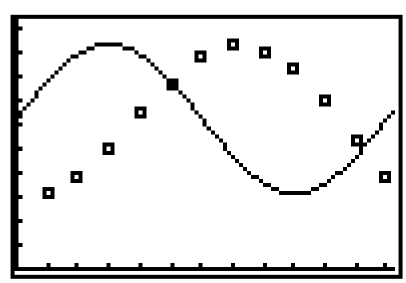
The graph of our Y1 is also centered on the *x*-axis...although it shouldn't be! It should be centered halfway between the highest and lowest temperatures. That is, the graph should be centered over the horizontal line shown at right.

Find the equation of that horizontal line. [Turn to the last page to write down your answer.]



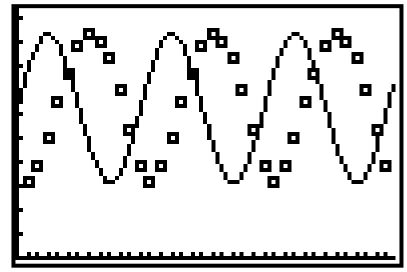
Once you have that line dialed in, you know by which value you need to translate your curve vertically! Adjust your function in Y1 (it should now be in the form $y = a\sin(kx) + d$) and re-graph it to get the view at right.

Now we have the right size and shape, but the curve is out of sync with our data. We'll address this next.



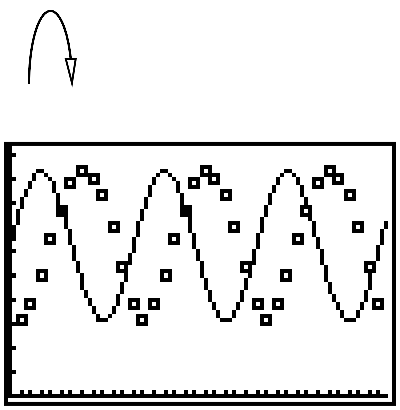
[e] FYI...The origin of the name "sine" actually comes from the Latin "sinus" or "a bay".

Task 5 (4 pts):

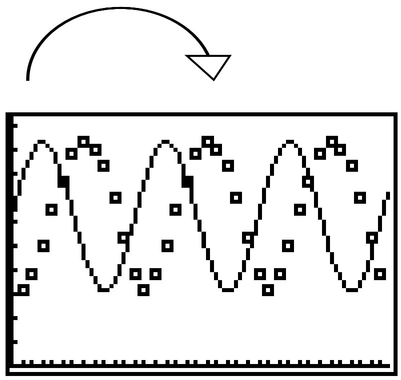


Phase shift is what it's called in trig, but when you studied it back in MTH 111, you called it *horizontal translation*. Whatever we called it, we need to slide our curve one way or the other. If I zoom out, you can see that our periodic sine curve is always a little ahead (or a little behind) our data curve. A horizontal translation will take care of this nicely.

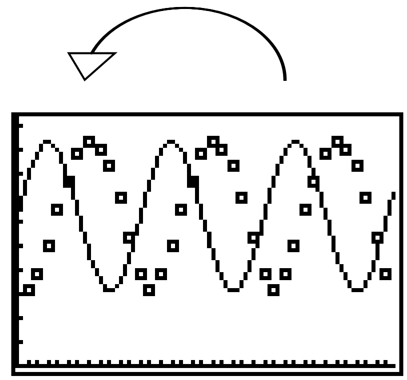
But which way to shift? Do we shift right, or left? Well, because of the periodicity of the sine curve, it doesn't matter! We can make, for example, any of these shifts:



<u>Choice 1</u>: Make the leftmost high point coincide with the closest data high point

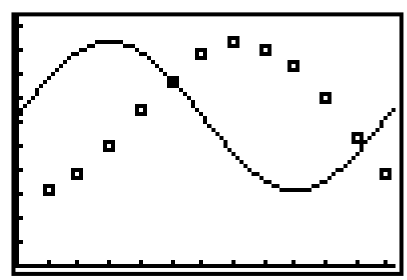


<u>Choice 2</u>: Make the leftmost high point coincide with the next data high point



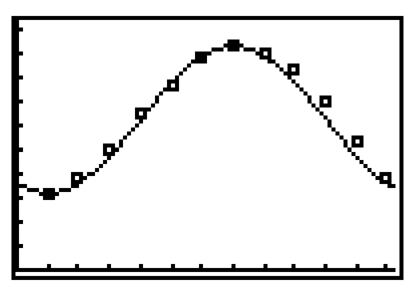
<u>Choice 3</u>: Make the rightmost high point coincide with the leftmost data high point

Since the sine curve is periodic, any of these will get us a correct equation.



I suggest that we focus on Choice 1, since it's the simplest; it'll get us in sync with the data with the smallest possible horizontal translation. Going back to your one-year graph, we can analyze how much we have to translate by looking at the high points of the data and of the curve.

[Turn to the last page to write down your answers.]



Now use your answers to apply a phase shift to your equation. You should now be in the form $y = a\sin(k(x-b)) + d$. Then graph that function, and watch it mesh with your data!

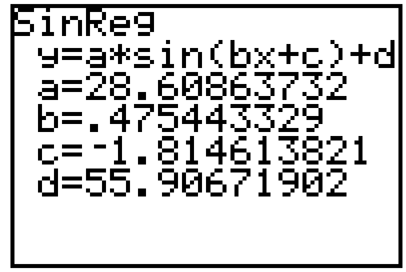
Now that you've fit the curve to the data, I'd like to see that equation:

Task 6 (3 pts): Write the equation you have in Y1...that is, the function $f(x)$ that gives the temperature ***f*** at time ***x***. Don't round any of your values (leave π's and decimals in there).
[Turn to the last page to write down your answer.]

Now, your TI can actually do a regression to find what it calls a "best-fit" curve[f]. Let's do this now, and compare its best-fit curve with our curve...

1. Press 2nd, MODE to quit to the homescreen.
2. Press STAT, right arrow, up arrow, ENTER to select the SinReg command.
3. Press VARS, right arrow, ENTER, down arrow, ENTER, ENTER to run the regression.

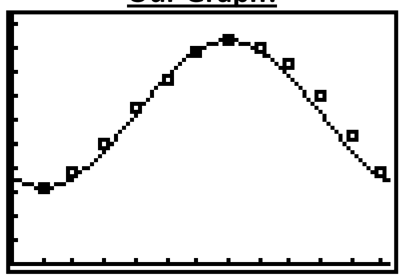
Your TI will think for a few moments, and then return the values shown at right. Notice that the TI uses the general from of the trigonometric equation $y = a\sin(bx + c) + d$, which ties in the vertical and horizontal translations, as well as the amplitude and period changes.



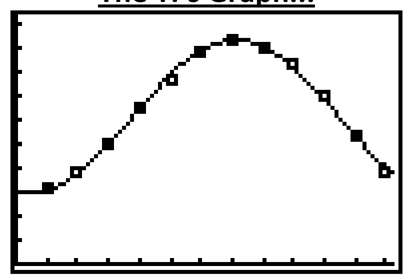
Now, let's take a look at how this graph looks with our data...

The graph at left is our graph, and the graph at right is the best-fit curve found by the TI...sure, it fits a ***little*** better over 12 months, but look at what happens when we extend the data out for two and then three years...

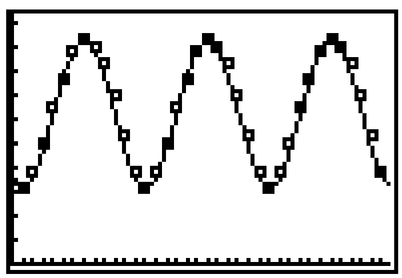
Our Graph!

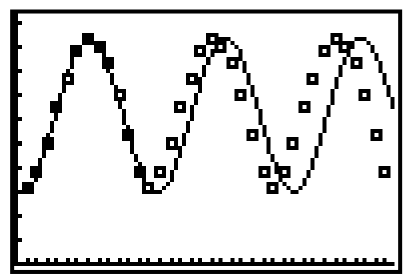


The TI's Graph...



Our extended graph actually fits the data better over time! Yay! You beat the TI! Can you see why?





Task 7 (3 pts): Why is the TI's graph skewed after 1 year, while ours stays consistently in line with our data? [Turn to the last page to write down your answer.]

Task 8 (3 pts): For the same Chicago temperature data, find the equation of a cosine curve that fits the data.[g] [Turn to the last page to write down your answer.]

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[f] You may have done a *linear* regression in the past. Your TI uses what is called a "least-squares iterative" method..."least squares" is a statistical idea, and "iterative" means that you give it a starting guess, and it runs from there. If you take a statistics class at some time, you'll study this.

[g] Hints: You can't use your TI's to do it automatically, because the TI only has SinReg. However, you ***can*** use the same amplitude, period and vertical translation you found before. You'll only have to adjust the horizontal translation.

Task 9 (6 pts): Here is Bend's monthly temperature data, from the Chamber of Commerce (http://bendchamber.org/weather/#temp):

Month	Average High Temperature
January	41
February	46
March	50
April	57
May	65
June	73
July	82
August	80
September	74
October	63
November	49
December	43

Repeat the process you did for Chicago to come up with an equation of the form $f(x) = a\sin\big(k(x-b)\big) + d$ for Bend's monthly highs.

Then use the regression on your calculator to come up with another (slightly different) version of this sine function.

[Turn to the last page to write down your answers.]

Lab 4 Answer Sheet

Task 1:
What is the amplitude ***a*** of our curve (don't round)? a =_______

Task 2:
What is the ***k***– value of our curve (don't round)? k = _______

Task 4:
The equation of that horizontal line is $f(x)$ =_______

Task 5:
The high point of your curve is at what x-value? ______

The high point of the data is at what x-value? ______

So how far do we have to shift the graph, and in what direction? ______ units to the ____________

Task 6:
$f(x) =$ ____

Task 7:
Why is the TI's graph skewed after 1 year, while ours stays consistently in line with our data?

Task 8:
$f(x) =$ ____

Task 9:

Your version: $f(x) =$ ____

Calculator's version (round values to three places):

$f(x) =$ ____

Grading Rubric:
For Tasks 1, 2, and 4, each answer is worth 2 points and you'll get 1 point if you make a small math error.
For Task 5, each answer is worth 1 point (all or nothing).
For Tasks 6 and 8, you'll earn 2 points if you make a small error, and 1 if you make a more major error but still show some understanding of the problem.
For Task 7, you'll earn 2 points if your answer basically contains the right reason but is unclearly stated, and 1 point if your answer has some valid mathematical basis but doesn't really address this question.
For Task 9, the first equation is worth 4 points, 1 for each constant. The second is worth 2 points and you'll get 1 point if you make a small math error.