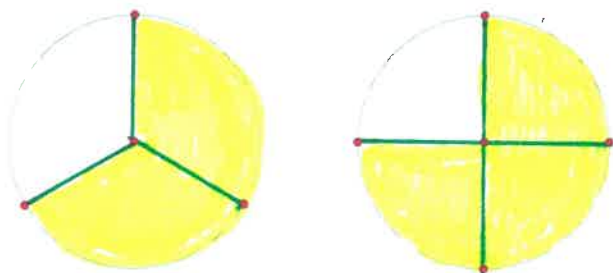


We've already done most of the work necessary to be able to add and subtract fractions. The most important thing to remember is that when adding and subtracting fractions, we need to be working with same size pieces or common denominators.

Let's do one more problem with pictures, to make certain you have a way to visualize the need for common denominators.

Add $\frac{2}{3} + \frac{3}{4}$

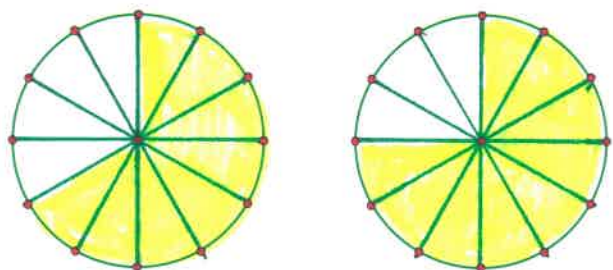
Shade in $\frac{2}{3}$ of the first circle and $\frac{3}{4}$ of the second circle.



Use this space to record what this problem would look like if done without the pictures.

$$\begin{array}{r} \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12} \\ + \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12} \\ \hline \frac{17}{12} = 1\frac{5}{12} \end{array}$$

Shade in $\frac{8}{12}$ of the first circle and $\frac{9}{12}$ of the second circle.



Think about how you know that you need to divide the circle into 12 pieces in order for this to work.

When you add $\frac{2}{3} + \frac{3}{4}$ you get an improper fraction or a number greater than 1. In algebra, we often leave fractions as improper, but with just numbers, writing $1\frac{5}{12}$ is often easier to wrap your head around than saying $\frac{17}{12}$. You should be comfortable with both!

When adding fractions, you can use any common multiple as the common denominator, however, your problem will probably be easier to work with if you use the lowest common denominator.

Here are a few problems to practice:

$$\begin{array}{r} \frac{3}{5} + \frac{7}{8} \\ \frac{24}{40} + \frac{35}{40} \\ \frac{59}{40} = 1\frac{19}{40} \end{array}$$

$$\begin{array}{r} \frac{5}{12} + \frac{7}{18} \\ \frac{15}{36} + \frac{14}{36} \\ \frac{29}{36} \end{array}$$

$$\begin{array}{r} \frac{1}{6} + \frac{3}{8} \\ \frac{4}{24} + \frac{9}{24} \\ \frac{13}{24} \end{array}$$

$$\begin{array}{r} \frac{3}{16} + \frac{7}{30} \\ \frac{45}{240} + \frac{56}{240} \\ \frac{101}{240} \end{array}$$

What about adding mixed numbers?

$$2\frac{3}{8} + 1\frac{1}{8}$$

You don't need to learn anything new to add mixed numbers, but you may need to unlearn something you may have been taught before, or take your brain off of auto-pilot.

When we multiplied and divided mixed numbers, we changed them to improper fractions, because they are easier to multiply and divide that way, but that isn't the case for addition and subtraction.

Looking at the pictures, it should be pretty clear that $2\frac{3}{8} + 1\frac{1}{8} = 3\frac{4}{8} = 3\frac{1}{2}$. Add the wholes and add the parts.



But many people change the mixed numbers into improper fractions, either because that's what you do for multiplication or because that is what they were taught. Changing the mixed numbers into improper fractions makes the picture look like...



$$2\frac{3}{8} + 1\frac{1}{8} = \frac{19}{8} + \frac{9}{8} = \frac{28}{8}$$

This isn't helpful! Then you still have to change $\frac{28}{8}$ to $3\frac{1}{2}$.

So to recap... When adding (and subtracting) mixed numbers, you still have to find common denominators for the fractional parts, but you don't need to change mixed numbers to improper fractions.

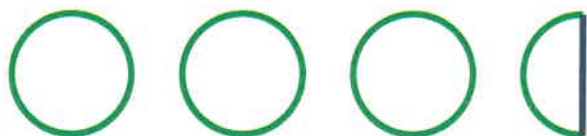
If you insist on doing this, you should still get the correct answer, but it will take you a lot longer, especially if it is a problem like $421\frac{3}{8} + 421\frac{5}{8}$ ☹.

What about **Subtracting Fractions**?

Most subtraction of fractions is just like addition. Find a common denominator then subtract, instead of add.

For example

$$3\frac{1}{2} - 1\frac{1}{4}$$



Here are $3\frac{1}{2}$ circles. Cross out $1\frac{1}{4}$.

You probably split the $\frac{1}{2}$ into two parts, which is just like making $\frac{1}{2} = \frac{2}{4}$. This is just what you do when you find common denominators. You can then cross out $1\frac{1}{4}$, leaving $2\frac{1}{4}$. Or...

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{2}{4} \\ -1\frac{1}{4} = -1\frac{1}{4} \\ \hline 2\frac{1}{4} \end{array}$$

Here are those same $3\frac{1}{2}$ circles. This time cross out (or subtract) $1\frac{3}{4}$.



This is a little trickier, but think about what you would do if there were $3\frac{1}{2}$ pizzas on the table and someone told you you could take $1\frac{3}{4}$ pizzas. I'm guessing you wouldn't hesitate to cut one of the whole pizza's into 4 pieces, while also cutting the half pizza into two slices.



By finding common denominators, the problem that was $3\frac{1}{2} - 1\frac{3}{4}$ becomes $3\frac{2}{4} - 1\frac{3}{4}$. It is then that you can see that subtracting $1\frac{3}{4}$ is a problem when there are only 2 pieces. Cutting one of the whole pizzas into 4 pieces makes the problem $2\frac{6}{4} - 1\frac{3}{4}$, which is no longer much of a problem.

The whole problem written out might look like...

$$\begin{array}{r}
 3\frac{1}{2} = 3\frac{2}{4} = 2\frac{6}{4} \\
 -1\frac{3}{4} = -1\frac{3}{4} = -1\frac{3}{4} \\
 \hline
 1\frac{3}{4}
 \end{array}$$

The one other type of fraction subtraction problem that sometimes causes problems is when you are asked to subtract a fraction or mixed number from a whole number like $5 - 1\frac{2}{3}$. Remember that in this situation $5 = 4\frac{3}{3}$, so you can

rewrite $5 - 1\frac{2}{3}$ as $4\frac{3}{3} - 1\frac{2}{3}$.

Here are a few problems to practice. Remember to always simplify your final answers.

$$\begin{array}{r}
 5\frac{2}{3} + 2\frac{3}{4} \\
 5\frac{8}{12} + 2\frac{9}{12} \\
 7\frac{17}{12} = 8\frac{5}{12}
 \end{array}$$

$$\begin{array}{r}
 3445\frac{2}{3} + 3442\frac{3}{4} \\
 3445\frac{8}{12} + 3442\frac{9}{12} \\
 6887\frac{17}{12} = 6888\frac{5}{12}
 \end{array}$$

$$\begin{array}{r}
 5\frac{3}{4} - 2\frac{2}{3} \\
 5\frac{9}{12} - 2\frac{8}{12} \\
 3\frac{1}{12}
 \end{array}$$

$$\begin{array}{r}
 5\frac{2}{3} - 2\frac{3}{4} \\
 5\frac{8}{12} - 2\frac{9}{12} \\
 4\frac{20}{12} - 2\frac{9}{12} \\
 2\frac{11}{12}
 \end{array}$$

$$\begin{array}{r}
 24 - 8\frac{3}{4} \\
 23\frac{4}{4} - 8\frac{3}{4} \\
 15\frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 5\frac{7}{32} + 9\frac{11}{18} \\
 5\frac{63}{288} + 9\frac{176}{288} \\
 14\frac{239}{288}
 \end{array}$$

$$\begin{array}{r}
 \frac{12}{17} + 32 \\
 32\frac{12}{17}
 \end{array}$$

$$\begin{array}{r}
 17\frac{2}{9} - 12\frac{13}{24} \\
 17\frac{16}{72} - 12\frac{39}{72} \\
 16\frac{88}{72} - 12\frac{39}{72} \\
 4\frac{49}{72}
 \end{array}$$

$$\begin{array}{r}
 9\frac{2}{5} + 3\frac{1}{6} \\
 9\frac{12}{30} + 3\frac{5}{30} \\
 12\frac{17}{30}
 \end{array}$$