Lab 5: Probability and Integration

In this lab you will do several problems that will help you understand how integrals (including improper ones!) relate to the concept of probability. Do the problems neatly in pencil on a **separate** piece of paper and staple your pages. Clearly lay out your work using proper notation and circle/highlight/box your final answer. **Problems with just an answer and no work will not receive credit.** You are encouraged to work groups of 2 to 4 people. If you do work with more than one person, you only need to hand in one lab write-up per group; make sure you put everyone's name on it. This lab is worth 40 points. This is due next **Wednesday, August 20.**

A **probability density function (pdf)** $f(x)$ is defined in the following way: if you have some quantity that can take on values within some interval of real numbers, then the probability that a randomly chosen value is between *a* and *b* is given by $\int_a^b f(x)\ dx$.

For example, if the quantity is heights of women in inches, and 65% of women are between 60 and 68 inches tall, then the pdf would satisfy $\int_{60}^{68} f(x)\ dx = 0.65$. A *normal distribution* has a pdf that is a bell curve. You can read more about that in section 8.5 if you are interested.

1. (5 pts) Any pdf must satisfy the following two criteria. Verbally explain why.

- $f(x) \geq 0$ for all *x*
- $\int_{-\infty}^{\infty} f(x)\ dx = 1$

2. (15 pts) The following pdf models wait times in minutes for calls to be answered by a certain company's representative: $f(t) = \begin{cases} 0 & \text{if } t < 0 \\ 0.1e^{-0.1t} & \text{if } t \geq 0 \end{cases}$

a) Verify that this function satisfies the second criterion from problem 1.
b) What is the probability that a call is answered in the first two minutes?
c) What is the probability that a customer waits more than 10 minutes to be answered?
d) The **mean** (average) value of a quantity with a pdf f is given by $\int_{-\infty}^{\infty} x\, f(x) dx$. What is the average wait time here?
e) The **median** value of a quantity with a pdf f is the number *m* such that $\int_m^{\infty} f(x) = 0.5$. What is the median wait time here?

3. (10 pts) The following pdf models times it takes students to complete a final exam, in hours:

$$f(t) = \begin{cases} \frac{t^3}{4} & \text{if } 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Verify that this function satisfies the second criterion from problem 1.
b) How long do you think the final exam period is, and why?
c) What percentage of students are still left after an hour and a half?
d) What is the average time it takes a student to complete the exam?

4. (10 pts) Only one of the following functions makes sense as a pdf representing the time (in minutes, starting from $t = 0$) that it takes for the next customer to walk into a store. The others don't, either because they don't satisfy the criteria for pdf's, or they do satisfy the criteria but don't make sense for this particular scenario. Identify which one makes sense, and for each of the others, explain why they don't, particularly noting if they fail the pdf criteria.

1) $f(t) = \begin{cases} \cos t & \text{if } 0 \le t \le 2\pi \\ e^{t-2\pi} & \text{if } t > 2\pi \end{cases}$

2) $f(t) = 3e^{-3t}$ for $t \ge 0$

3) $f(t) = e^{-3t}$ for $t \ge 0$

4) $f(t) = \frac{1}{4}$ for $0 \le t \le 4$