Lab 5 Key

1. For the first criterion, an event can't have a negative probability. For the second, the interpretation is that the probability the quantity is in $(-\infty, \infty)$ is 100%, and that's always true no matter what we're measuring.

2. a) $\int_{-\infty}^{\infty} f(t)\,dt$ is really $\int_0^{\infty} 0.1e^{-0.1t}\,dt$ here, since $\int_{-\infty}^{0} f(t)\,dt = 0$. So we calculate $\lim_{b\to\infty} \int_0^b 0.1e^{-0.1t}\,dt = \lim_{b\to\infty} -e^{-0.1t}\Big|_0^b$

$$= \lim_{b\to\infty} -e^{-0.1b} - (-e^{-0.1(0)}) = \lim_{b\to\infty} -e^{-0.1b} + 1 = 1.$$

b) $\int_0^2 0.1e^{-0.1t}\,dt = -e^{-0.1t}\Big|_0^2 = -e^{-0.1(2)} + 1 \approx 0.18$, or $\boxed{18\%}$

c) We can either calculate $\int_{10}^{\infty} 0.1e^{-0.1t}\,dt$ or $1 - \int_0^{10} 0.1e^{-0.1t}\,dt$
I'll show both:

$$\int_{10}^{\infty} 0.1e^{-0.1t}\,dt = \lim_{b\to\infty} -e^{-0.1t}\Big|_{10}^{b} = \lim_{b\to\infty} -e^{-0.1b} - (-e^{-0.1(10)})$$

$= e^{-1} \approx 0.37$ or $\boxed{37\%}$

or

$$1 - \int_0^{10} 0.1e^{-0.1t}\,dt = 1 - (-e^{-0.1t})\Big|_0^{10} = 1 - (-e^{-1} - (-e^{0})) = 1 - (-e^{-1} + 1)$$
$$= e^{-1} \approx 0.37$$

d) $\int_{-\infty}^{\infty} t f(t)\,dt = \int_0^{\infty} t(0.1e^{-0.1t})\,dt$

This is an integration by parts problem: $u = t$ $v = -e^{-0.1t}$

$du = 1\,dt$ $dv = 0.1e^{-0.1t}\,dt$

So we have $\lim_{b\to\infty} \left(t(-e^{-0.1t})\Big|_0^b - \int_0^b -e^{-0.1t}\,dt \right)$

$$= \lim_{b\to\infty} \left(-be^{-0.1b} - 0 - (10e^{-0.1t})\Big|_0^b \right)$$

$$= \lim_{b\to\infty} -be^{-0.1b} - (10e^{-0.1b} - 10e^{-0.1(0)}) = 0 - (0 - 10) = \boxed{10\text{ min}}$$

e) $\int_m^\infty 0.1e^{-0.1t}\,dt = 0.5$

$\int_m^\infty 0.1e^{-0.1t}\,dt = \lim_{b\to\infty} -e^{-0.1t}\Big|_m^b = \lim_{b\to\infty} -e^{-0.1b} - (-e^{-0.1m}) = e^{-0.1m}$

So $e^{-0.1m} = 0.5$

$-0.1m = \ln 0.5$

$m \approx \boxed{6.9\text{ min}}$

3. a) $\int_{-\infty}^{\infty} f(t)\,dt = \int_0^2 t^3/4\,dt = t^4/16\Big|_0^2 = 16/16 - 0/16 = 1.$

b) 2 hours, since there is 0 probability a student will take more than 2 hrs.

c) $\int_{1.5}^2 t^3/4 = t^4/16\Big|_{1.5}^2 = 1 - 1.5^4/16 \approx \boxed{68\%}$

d) $\int_{-\infty}^{\infty} t\,f(t)\,dt = \int_0^2 t\cdot t^3/4\,dt$

$= \int_0^2 t^4/4\,dt = t^5/20\Big|_0^2 = 2^5/20 - 0 = \boxed{1.6\text{ hrs or }1\text{ hr }36\text{ min}}$

4. 1) This can't be a pdf since cos t has negative values on $[0, 2\pi]$ (the first criterion isn't met).

2) $\int_0^\infty 3e^{-3t}\,dt = \lim_{b\to\infty} -e^{-3t}\Big|_0^b = \lim_{b\to\infty} -e^{-3b} - (-e^0) = 1$

So it could be a pdf & is the best option so far.

3) $\int_0^\infty e^{-3t}\,dt = \lim_{b\to\infty} -1/3\,e^{-3t}\Big|_0^b = \lim_{b\to\infty} -1/3\,e^{-3b} - (-1/3\,e^0) = 1/3$

so it doesn't meet the 2nd criterion.

4) $\int_0^4 1/4\,dt = 1/4\,t\Big|_0^4 = 1$ so it could be a pdf. However, it says there is 0 probability it will be more than 4 minutes until the next customer arrives, which is unreasonable

Choice 2 makes the most sense.