Lab 4 Key

1. $\int \frac{x^3+x+1}{x^2}\,dx$

Since the denominator is a monomial, let's divide through by it:

$\int x + \frac{1}{x} + \frac{1}{x^2}\,dx = \int x + x^{-1} + x^{-2}\,dx = \boxed{\frac{x^2}{2} + \ln|x| - x^{-1} + C}$

2. $\int x\sqrt{4-x^2}\,dx$

Let $u = 4-x^2$; then $du = -2x\,dx \rightarrow -\frac{1}{2}du = x\,dx$

Then we have $\int \sqrt{u}\cdot -\frac{1}{2}du = -\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$

$= -\frac{1}{3}u^{3/2} + C = \boxed{-\frac{1}{3}(4-x^2)^{3/2} + C}$

3. $\int \frac{x+1}{x^2-3x+2}\,dx$

The denominator factors as $(x-1)(x-2)$, so we have

$\frac{x+1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} \rightarrow x+1 = A(x-2) + B(x-1)$

$x+1 = (A+B)x - 2A - B$

$A + B = 1$

$-2A - B = 1$

$-A = 2$

$A = -2$

$-2 + B = 1$

$B = 3$

So we have $\int \frac{-2}{x-1} + \frac{3}{x-2}\,dx$

$= \boxed{-2\ln|x-1| + 3\ln|x-2| + C}$

4. $\int \frac{\sec^2 x \tan^2 x}{\sqrt{9-\tan^2 x}}\,dx$

Let $u = \tan x$, so $du = \sec^2 x\,dx$. Then we have $\int \frac{u^2}{\sqrt{9-u^2}}\,du$

This is in the form of Table Line 34, where $a=3$.

So we have $-\frac{u}{2}\sqrt{9-u^2} + \frac{9}{2}\sin^{-1}\left(\frac{u}{3}\right) + C$

$= \boxed{\frac{-\tan x\sqrt{9-\tan^2 x} + 9\sin^{-1}\left(\frac{\tan x}{3}\right)}{2} + C}$

5. $\int x(x-10)^{10}\,dx$

Let $u = x-10$, so $du = dx$, and also $x = u+10$. So we have

$\int (u+10)u^{10}\,du = \int u^{11} + 10u^{10}\,du = \frac{u^{12}}{12} + \frac{10u^{11}}{11} + C$

$= \boxed{\frac{(x-10)^{12}}{12} + \frac{10(x-10)^{11}}{11} + C}$

6. $\int x^2 e^{2x}\,dx$

Since derivatives of x^2 are simpler & antiderivatives of e^{2x} are no more complicated, we'll integrate by parts.

$u = x^2$ $\quad [v = \frac{1}{2}e^{2x}]$

$[du = 2x\,dx]$ $\quad dv = e^{2x}\,dx$

So we have $x^2 \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} \cdot 2x\,dx = \frac{1}{2}x^2e^{2x} - \int xe^{2x}\,dx$

$u = x$ $\quad [v = \frac{1}{2}e^{2x}]$

$[du = dx]$ $\quad dv = e^{2x}\,dx$

So we have $\frac{1}{2}x^2e^{2x} - \left(x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right)$

$= \boxed{\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$

* You could also use Table Line 97

7. $\int \frac{x^4}{\sqrt{x^{10}-2}}\,dx$

There are several table forms with $\sqrt{u^2-a^2}$, so let's try $u = x^5$, so $du = 5x^4\,dx \rightarrow \frac{1}{5}du = x^4\,dx$. Then we have $\int \frac{1}{\sqrt{u^2-2}} \cdot \frac{1}{5}\,du = \frac{1}{5}\int \frac{1}{\sqrt{u^2-2}}\,du$. This is in the form of Table Line 43 with $a=\sqrt{2}$. So we have $\ln|u+\sqrt{u^2-2}|+C$

$= \boxed{\ln|x^5+\sqrt{x^{10}-2}|+C}$

8. $\int \frac{1}{x^2+4x+5}\,dx$

Since x^2+4x+5 doesn't factor, we should complete the square:

$x^2+4x+4-4+5 = (x+2)^2+1$

So we have $\int \frac{1}{(x+2)^2+1}\,dx$ $\quad u = x+2 \rightarrow du = dx$ gives us

$\int \frac{1}{u^2+1}\,du = \tan^{-1}u + C = \boxed{\tan^{-1}(x+2)+C}$

9. $\int \left(\frac{x}{3}+\frac{3}{x}\right)^2 dx$

Since this is easy to FOIL, let's check that out first:

$\int \frac{x^2}{9} + \frac{3x}{3x} + \frac{3x}{3x} + \frac{9}{x^2}\,dx = \int \frac{1}{9}x^2 + 2 + 9x^{-2}\,dx$

$= \boxed{\frac{1}{27}x^3 + 2x - 9x^{-1} + C}$

10. $\int e^{5x} \cos(2x)\, dx$

This is exactly Table Line 99 with $u = x$, $a = 5$, and $b = 2$.

So we have $\dfrac{e^{5x}}{5^2 + 2^2}(5 \cos 2x + 2 \sin 2x) + C$

$$= \boxed{\frac{5}{29} e^{5x} \cos 2x + \frac{2}{29} e^{5x} \sin 2x + C}$$