Lab 3 Key

1. For $\int 1/1+x^2 \, dx$, the most apparent choice for substitution is $u = 1 + x^2$, in which case $du = 2x\,dx$. Since there isn't an additional factor of $x$ in the integrand, that won't work. And there aren't any other options for $u$ that turn this into something we know how to integrate.
(In fact, no special method is needed: it's $\tan^{-1}x + C$.)
For $\int x/1+x^2 \, dx$, when we let $u = 1+x^2$ so $du = 2x\,dx$, we do see a constant multiple of $du$ in the integrand: $x\,dx = 1/2\,du$. So we have $\int 1/u \cdot 1/2\,du = 1/2 \ln|u| + C$
$= 1/2 \ln|1+x^2| + C$.

2. with substitution:
let $u = 2x^2 + 3x$, so $du = 4x+3\,dx$
then we have $\int u^2\,du = u^3/3 + C = (2x^2+3x)^3/3 + C$
by multiplying out:
$(2x^2+3x)^2(4x+3) = (4x^4+12x^3+9x^2)(4x+3)$
$= 16x^5 + 48x^4 + 36x^3 + 12x^4 + 36x^3 + 27x^2$
so the integral is $\int 16x^5 + 60x^4 + 72x^3 + 27x^2\,dx$
$= 8/3\,x^6 + 12x^5 + 18x^4 + 9x^3 + C$
do they match?
multiplying out our first result, we have
$(2x^2+3x)(2x^2+3x)(2x^2+3x)/3 + C = (4x^4+12x^3+9x^2)(2x^2+3x)/3 + C$
$= 8x^6 + 24x^5 + 18x^4 + 12x^5 + 36x^4 + 27x^3/3 + C$
$= 8/3\,x^6 + 12x^5 + 18x^4 + 9x^3 + C$ YES!

3. The most "inside" function is $3+\ln x$, so let $u=3+\ln x$, so $du = 1/x\, dx$. To "back substitute" for $2-\ln x$, note $\ln x = u-3$, so $2-\ln x = 2-(u-3) = 5-u$. Then we have $\int u^2(5-u)/4\, du = \int 5/4\, u^2 - 1/4\, u^3\, du = 5/12\, u^3 - 1/16\, u^4 + C$ $= 5/12\,(3+\ln x)^3 - 1/16\,(3+\ln x)^4 + C$.

4. You might first try $u = t+e^t$, so $du = 1+e^t\, dt$, but that doesn't lead anywhere. However, $e^{t+e^t} = e^t e^{e^t}$, so we have $\int e^t e^{e^t}\, dt$. This indicates a substitution of $u=e^t \to du = e^t dt$ Then we have $\int e^u\, du = e^u + C = e^{e^t} + C$.

5. For the integral $\int_0^3 x\, f(x^2)\, dx$, the obvious substitution is $u=x^2$, so $du = 2x\, dx \to 1/2\, du = x\, dx$. Also $x=0,3$ means $u=0,9$. So we have $\int_0^9 f(u)\cdot 1/2\, du = 1/2 \int_0^9 f(u)\, du$ But we know $\int_0^9 f(x)\, dx = 4$, and the variable name is irrelevant. So $1/2 \int_0^9 f(u)\, du = 1/2(4) = 2$.