Lab 2: Definite Integrals and the Fundamental Theorem

In this lab you will do several problems that will help you practice working with and applying definite integrals. Do the problems neatly in pencil on a **separate** piece of paper and staple your pages. Clearly lay out your work using proper notation and circle/highlight/box your final answer. **Problems with just an answer and no work will not receive credit.** You are encouraged to work groups of 2 to 4 people. If you do work with more than one person, you only need to hand in one lab write-up per group; make sure you put everyone's name on it. This lab is worth 40 points. This is due next **Wednesday, July 9.**

1. (10 pts) Let $f(x) = \begin{cases} 0 & if\ x < 0 \\ x & if\ 0 \le x \le 1 \\ 2 - x & if\ 1 < x \le 2 \\ 0 & if\ x > 2 \end{cases}$ and let $g(x) = \int_0^x f(t)dt$.

a) Find a piecewise expression for $g(x)$ that does **not** involve integrals.
b) Sketch the graphs of f and g.
c) Where is f differentiable? Where is g differentiable? Explain.

2. (8 pts) Calculate $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2i}{n} - \left(\frac{i}{n}\right)^2\right] \frac{1}{n}$ by first figuring out what definite integral (from 0 to 1) this limit of right-hand sums is equal to, and then using FTC2 to evaluate it by hand.

Evaluate the integrals in the remaining problems by hand using FTC2 as well, though you may check your answers with your calculator's integration tool. (Of course, you may also use the calculator to actually plug your limits of integration into the antiderivative, if necessary.)

3. (10 pts) The integral of a velocity function (rate of change of position) gives you total change in position. Similarly, any integral of a function which represents the rate of change of a quantity will give you the total change in that quantity.

   After WWII, the birth **rate** in Western countries increased dramatically. Suppose the number of babies born (in millions per year) was given by $b(t) = 5 + 2t, 0 \le t \le 10$, where t is in years since the end of the war.

a) How many babies were born in the 10 years after the war?
b) How many years did it take until the total number of babies born after the war was 14 million?

4. (10 pts) Continuing with the previous theme: the tide removes sand from a beach at a **rate** modeled by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$, where t is in hours since midnight and the rate is in ft^3/hr. How much sand was removed by 6am?
Note that this antiderivative is a more challenging one to find than we've addressed so far. Trial and error is a perfectly acceptable way to find it. However you find it, please document your thought process.

5. (2 pts) The rate at which rain falls in inches/hour is $R(t)$, where t is in hours since midnight, which is when the rain started falling. You are not required to show work for this problem.
a) The rain stops falling at 10am. Express the total rainfall, in inches, as an integral.
b) Interpret $\int_2^5 R(t)\ dt$ in terms of this scenario.