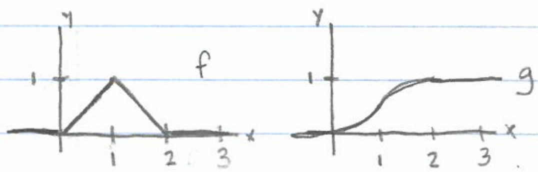
Lab 2 Key

1.a) For $x<0$, there is no area under the curve, so $g(x)=0$.
For $0 \le x \le 1$, $g(x) = \int_0^x t\,dt = t^2/2 \big|_0^x = x^2/2$
For $1 < x \le 2$, $g(x) = \int_0^1 t\,dt + \int_1^x 2-t\,dt = t^2/2 \big|_0^1 + (2t - t^2/2)\big|_1^x$
$= 1/2 + 2x - x^2/2 - (2 - 1/2) = -x^2/2 + 2x - 1$
Since f only has non-zero area under the curve on $[0,2]$,
for $x>2$ $g(x) = \int_0^2 f(t)\,dt = t^2/2 \big|_0^1 + (2t - t^2/2)\big|_1^2$
$= 1/2 + [(4-2)-(2-1/2)] = 1$
So $g(x) = \begin{cases} 0 & \text{if } x<0 \\ x^2/2 & \text{if } 0 \le x \le 1 \\ -x^2/2 + 2x - 1 & \text{if } 1 < x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$

b) 

c) f is not differentiable at $x = 0, 1, \& 2$ since there are sharp corners... so it's differentiable on $(-\infty,0) \cup (0,1) \cup (1,2) \cup (2,\infty)$.
g is differentiable everywhere.

2. A right-hand sum approximation for an integral has the form $\sum_{i=1}^n f(x_i)\Delta x$. We have $\sum_{i=1}^n [2i/n - (i/n)^2]\, 1/n$.
This seems to indicate that $\Delta x = 1/n$. The fact that i/n shows up in the sum corroborates this: $[0,1]$ is being divided into intervals $[0, 1/n], [1/n, 2/n]$, etc. That is, $x_i = i/n$.
So $f(x_i) = 2(i/n) - (i/n)^2 = 2x_i - x_i^2$.
Then this limit is $\int_0^1 2x - x^2\,dx = x^2 - x^3/3 \big|_0^1 = 1 - 1/3 = 2/3$

3. a) $\int_0^{10} 5+2t\,dt = 5t+t^2 \Big|_0^{10} = 150$

So 150 million babies were born in Western countries in the 10 yrs after WWII.

b) We want x such that $\int_0^x 5+2t\,dt = 14$

$5t+t^2 \Big|_0^x = 14$

$5x+x^2 = 14$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x = \cancel{-7}, 2$

It took 2 years for 14 million babies to be born

4. We need to calculate $\int_0^6 2+5\sin\left(\frac{4\pi}{25}t\right) dt$.

So what is the antiderivative of $2+5\sin\left(\frac{4\pi}{25}t\right)$?

We can guess $2t-5\cos\left(\frac{4\pi}{25}t\right)$. But using the chain rule, its derivative is $2+5\sin(4\pi/25\,t)\cdot 4\pi/25$.

To get rid of that extra factor of $4\pi/25$, multiply by the reciprocal: $\int_0^6 2+5\sin(4\pi/25\,t)\,dt$

$= 2t - 125/4\pi\cos(4\pi/25\,t)\Big|_0^6$

$= (12-125/4\pi\cos 24\pi/25) - (0-125/4\pi\cos 0)$

$\approx 31.82\ ft^3$

5. a) $\int_0^{10} R(t)\,dt$

b) It tells us the total rainfall between 2am and 5am.