4. $f'(x) = x^3 \rightarrow f(x) = 1/4\, x^4 + C$ ... we just need C

f has tangent line $x + y = 0 \rightarrow y = -x$, which has slope -1.

So at that point, $f'(x) = -1$. But that means $x^3 = -1 \rightarrow x = -1$.

So there is a point with x-coordinate -1 that lies on both $f(x) = 1/4\, x^4 + C$ and $y = -x \rightarrow$ that means the point is $(-1, 1)$.

So $1 = 1/4\,(-1)^4 + C \rightarrow C = 3/4 \rightarrow$ $\boxed{f(x) = 1/4\, x^4 + 3/4}$

5. The antiderivative F must satisfy $F(0) = 7$ and $F'(x) = f(x)$ for all x. Specifically, $F'(0) = f(0) = 2$. So the slope of the tangent line to $F(x)$ at $x = 0$ is 2, and it passes through $(0, 7)$.

$y - 7 = 2(x - 0)$

$\boxed{y = 2x + 7}$ is the equation of the tangent line