Names KEY

Group Work 3: Proving Trig Identities

Verify each identity, showing each step explicitly. That means that at each step, you should be using *one* algebraic technique or *one* identity. You can either transform one side into the other, or transform both sides into the same simplified expression.

You will be graded on the *clarity and accuracy* of your work—i.e., is it both correct and easy to follow? **ADVICE: You may want to consider writing a draft on a separate page first.** Each problem is worth 6 points, for a total of 48 points.

1. $\frac{\cos u \sec u}{\tan u} = \cot u$

$$\text{LHS} = \frac{\cos u \cdot \frac{1}{\cos u}}{\tan u} = \frac{1}{\tan u} = \cot u = \text{RHS}$$

2. $\frac{1}{1-\sin^2 y} = 1 + \tan^2 y$

$$\text{LHS} = \frac{1}{\cos^2 y} = \sec^2 y = 1 + \tan^2 y = \text{RHS}$$

Note: There are usually many ways to prove an identity. It's okay if you didn't find the simplest way — I might not always have found it here either!

3. $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

$$\text{LHS} = \frac{1-\cos x}{\sin x} \cdot \frac{(1+\cos x)}{(1+\cos x)} = \frac{1-\cos x+\cos x-\cos^2 x}{\sin x(1+\cos x)}$$

$$= \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \frac{\sin^2 x \div \sin x}{\sin x(1+\cos x) \div \sin x} = \frac{\sin x}{1+\cos x} = \text{RHS}$$

4. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin\theta\cdot\sin\theta}{\cos\theta\cdot\sin\theta} + \frac{\cos\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} = \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} = \sec\theta\csc\theta$$

$$= \text{RHS}$$

5. $\frac{\sec x}{\sec x - \tan x} = \sec x\,(\sec x + \tan x)$

$$LHS = \frac{\frac{1}{\cos x}}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{1-\sin x}{\cos x}} = \frac{1}{\cos x} \cdot \frac{\cos x}{1-\sin x} = \frac{1}{1-\sin x}$$

$$RHS = \frac{1}{\cos x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} = \frac{1+\sin x}{\cos^2 x}$$

$$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1+\sin x}{(1+\sin x)(1-\sin x)} = \frac{1}{1-\sin x}$$

6. $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

$$LHS = (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$

$$= 1(\sin^2\theta - \cos^2\theta)$$

$$= \sin^2\theta - \cos^2\theta$$

$$= \sin^2\theta - (1 - \sin^2\theta)$$

$$= 2\sin^2\theta - 1$$

$$= RHS$$

7. $\frac{1}{\sec x+\tan x}+\frac{1}{\sec x-\tan x}=2\sec x$

$$\text{LHS} = \frac{1}{\sec x+\tan x}\frac{(\sec x-\tan x)}{(\sec x-\tan x)}+\frac{1}{\sec x-\tan x}\frac{(\sec x+\tan x)}{(\sec x+\tan x)}$$

$$= \frac{\sec x-\tan x+\sec x+\tan x}{\sec^2 x-\sec x\tan x+\sec x\tan x-\tan^2 x}$$

$$= \frac{2\sec x}{\sec^2 x-\tan^2 x}$$

$$= \frac{2\sec x}{1}$$

$$= \text{RHS}$$

8. $\frac{\cos^2 t+\tan^2 t-1}{\sin^2 t}=\tan^2 t$

$$\text{LHS} = \frac{\cos^2 t}{\sin^2 t}+\frac{\tan^2 t}{\sin^2 t}-\frac{1}{\sin^2 t}$$

$$= \cot^2 t+\frac{\frac{\sin^2 t}{\cos^2 t}}{\sin^2 t}-\csc^2 t$$

$$= \cot^2 t+\frac{\cancel{\sin^2 t}}{\cos^2 t}\cdot\frac{1}{\cancel{\sin^2 t}}-\csc^2 t$$

$$= \cot^2 t+\sec^2 t-\csc^2 t$$

$$= \cot^2 t+1+\tan^2 t-\csc^2 t$$

$$= \csc^2 t+\tan^2 t-\csc^2 t$$

$$= \tan^2 t$$

$$= \text{RHS}$$