Name KEY

Exam 1

You may use a calculator on this exam, but you should also show work if you hope to earn any partial credit for an incorrect answer. Point values are listed by each problem for a total of 100 points. **For problems where you show work, please circle or box your final answers** (if you're not given a blank to write them in). If you are unsure about what a question is asking, raise your hand or come up to my desk. Good luck!

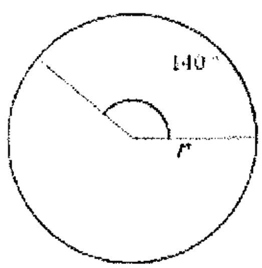
1. (2 pts) Convert $\frac{\pi}{5}$ to degrees.

$$\frac{\pi}{5}\cdot\frac{180}{\pi}=\frac{180}{5}=\boxed{36^\circ}$$

2. (2 pts) Write 75° in radians as a fraction of π (not a decimal).

$$\frac{75}{1}\cdot\frac{\pi}{180}=\frac{75\pi}{180}\begin{matrix}\div 5\\ \div 5\end{matrix}=\frac{15\pi}{36}\begin{matrix}\div 3\\ \div 3\end{matrix}=\boxed{\frac{5\pi}{12}}$$

3. (4 pts) Given that the shaded area of the sector enclosed by a central angle of 140° is 16 square inches, find the radius of the circle to the nearest hundredth (two places), and use appropriate units. The formula is $A=\frac{1}{2}\theta r^2$ where θ is in **radians.**



$$140^\circ=\frac{140}{1}\cdot\frac{\pi}{180}=\frac{140\pi}{180}\begin{matrix}\div 20\\ \div 20\end{matrix}=\frac{7\pi}{9}$$

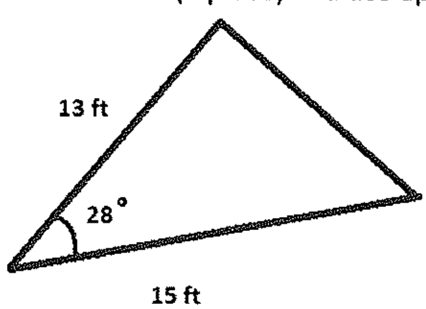
$A=16$, $\theta=7\pi/9$, so:

$$16=\frac{1}{2}\left(\frac{7\pi}{9}\right)r^2\rightarrow 16=\frac{7\pi}{18}r^2\rightarrow r^2\approx 13.096\ldots$$

$\cdot 18/7\pi \quad \cdot 18/7\pi$

$$r\approx\boxed{3.62\text{ in.}}$$

4. (4 pts) Find the area of the triangle below. The formula is $A=\frac{1}{2}ab\sin\theta$. Round to the nearest tenth (1 place) and use appropriate units.



$$A=\frac{1}{2}\cdot 13\cdot 15\cdot\sin(28^\circ)$$

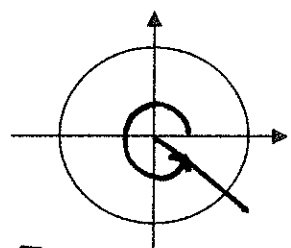
$$A\approx\boxed{45.8\text{ ft}^2}$$

5. (32 pts) Fill in the missing parts – blanks for parts (a) thru (d) – using exact values in the form of simplified radicals, fractions, or whole numbers - **no decimals.**

- When necessary: write the angle measure, θ, in **both radian and degrees** using only the **standard angles** in the intervals $[0, 2\pi)$ and $[0°, 360°)$;
- Identify the measure of the reference angle, $\bar{\theta}$, in **degrees** (N/A is a possible answer);
- **Clearly draw the standard angle in standard position** using the indicator curve with an arrow – **see (b)**;
- Label the ordered pair representing the point of intersection with the terminal side of the angle and the given unit circle (like in part b).

a)

$\theta = \frac{7\pi}{4} =$ 315°



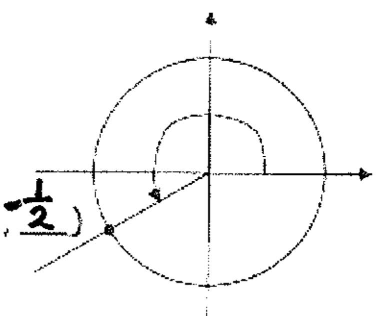
$\bar{\theta}=$ 45°

($\sqrt{2}/2$, $-\sqrt{2}/2$)

$\sin\theta =$ $-\sqrt{2}/2$, $\cos\theta =$ $\sqrt{2}/2$, $\tan\theta =$ -1

b)

$\theta =$ 210° $=$ $7\pi/6$



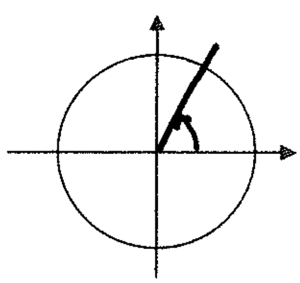
($\frac{-\sqrt{3}}{2}$, $-\frac{1}{2}$)

$\bar{\theta}=$ 30°

$\sin\theta =$ $-1/2$, $\cos\theta =$ $-\sqrt{3}/2$, $\tan\theta =$ $1/\sqrt{3}$

c) $\theta_c = -300°$ is coterminal with which standard angle?

$\theta=$ 60° $=$ $\pi/3$

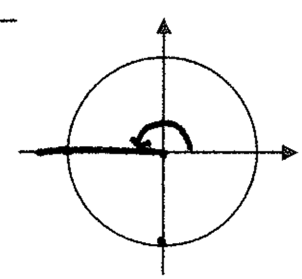


$\bar{\theta}=$ 60°

($\frac{1}{2}$, $\frac{\sqrt{3}}{2}$)

$\sin\theta =$ $\sqrt{3}/2$, $\cos\theta =$ $1/2$, $\tan\theta =$ $\sqrt{3}$

d) $\theta=$ 180° $=$ π



$\bar{\theta}=$ N/A

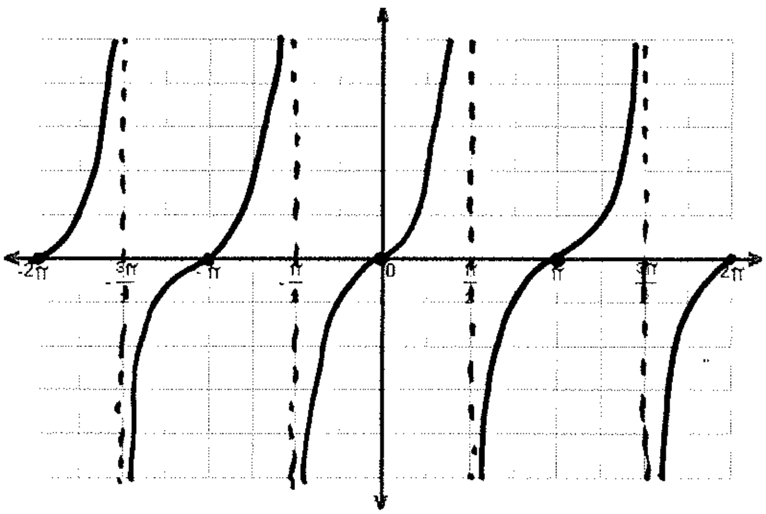
(-1 , 0)

$\sin\theta =$ 0, $\cos\theta = -1$, $\tan\theta =$ 0

6. (14 pts)
   a) Which of the three main trig functions (sine, cosine, or tangent) has a graph with asymptotes?

   tangent

   b) Sketch the graph of the function you chose, using dotted lines to represent asymptotes. Make sure that zeros and asymptotes are correctly placed. Sketch all the way from the left to the right.



   c) What is the domain?

   $\{x \mid x \neq \pi/2 + \pi n$, where n is an integer$\}$

   d) What is the range?

   $\{y \mid y$ is a real number$\}$ OR $(-\infty, \infty)$

   e) What is the period?

   π

7. (10 pts) Find the given trig functions of θ, given that $\sin\theta = \frac{8}{15}$ and θ is in quadrant II. ← only sin, csc are positive

$\cos(\theta) = \underline{-\frac{\sqrt{161}}{15}}$ $\tan(\theta) = \underline{-\frac{8}{\sqrt{161}}}$

$\cot(\theta) = \underline{-\frac{\sqrt{161}}{8}}$ $\sec(\theta) = \underline{-\frac{15}{\sqrt{161}}}$ $\csc(\theta) = \underline{\frac{15}{8}}$

$$\left(\frac{8}{15}\right)^2 + (\cos\theta)^2 = 1$$

$$\frac{64}{225} + (\cos\theta)^2 = 1$$

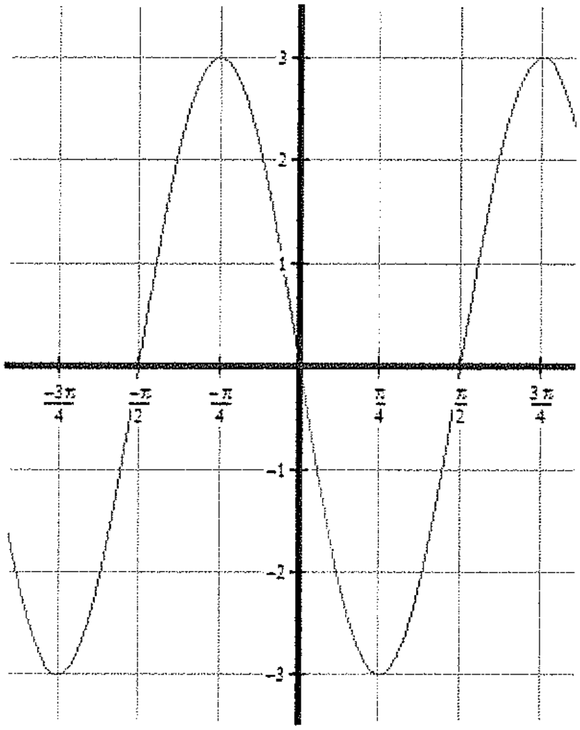
$$(\cos\theta)^2 = \frac{161}{225}$$

$$\cos\theta = \pm\sqrt{\frac{161}{225}}$$

$$\cos\theta = -\frac{\sqrt{161}}{15}$$

NOTE: because of periodicity, there are infinitely many possibilities for each answer here

8. (12 pts) Give a possible equation for each graph (you may use your calculator as a check! All numbers should be in exact form, no decimals.



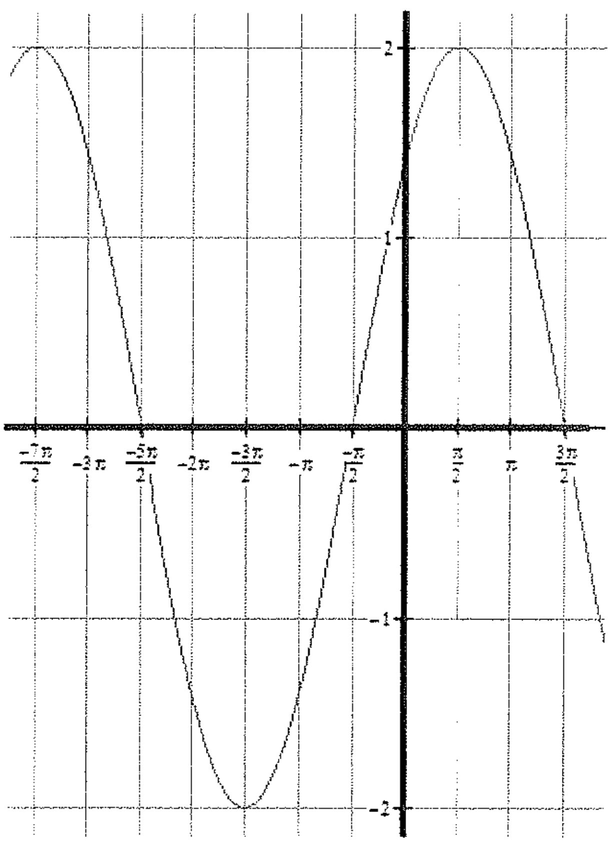
period $= \pi = \frac{2\pi}{k}$ so $k=2$

amplitude $= a = 3$

this is a sine graph reflected over the x-axis but not shifted or a cosine graph shifted $\pi/4$ left (so $b=-\frac{\pi}{4}$)

some possibilities:

$$\boxed{\begin{array}{l} y=-3\sin(2x) \\ y=3\cos\left(2\left(x+\frac{\pi}{4}\right)\right) \end{array}}$$



period $= 4\pi = \frac{2\pi}{k}$ so $k=\frac{1}{2}$

amplitude $= a = 2$

if sine, $b=-\frac{\pi}{2}$ (left shift)

if cosine, $b=\frac{\pi}{2}$ (right shift)

some possibilities:

$$y=2\sin\left(\tfrac{1}{2}\left(x+\tfrac{\pi}{2}\right)\right)$$

$$y=2\cos\left(\tfrac{1}{2}\left(x-\tfrac{\pi}{2}\right)\right)$$

9. (20 pts) For the given functions, sketch each graph, showing at least two full periods. Clearly **label the scale** on both axes and clearly and accurately plot the **5 key points**. Identify the period, amplitude, and phase shift. Use radians on your *x*-axis, not degrees.

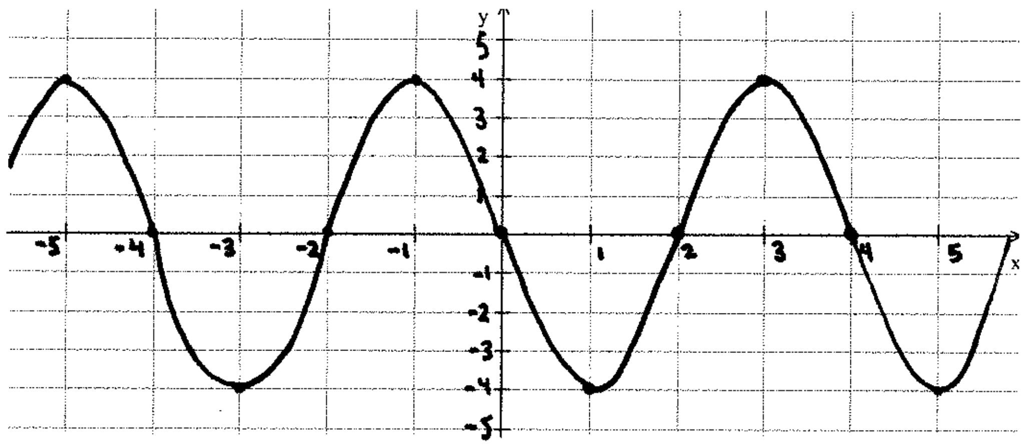
reflection over x-axis

a) $y = -4\sin\left(\frac{\pi}{2}x\right)$

Amplitude = 4

Period = $2\pi / \frac{\pi}{2} = 2\pi \cdot \frac{2}{\pi} = 4$

Phase shift = 0

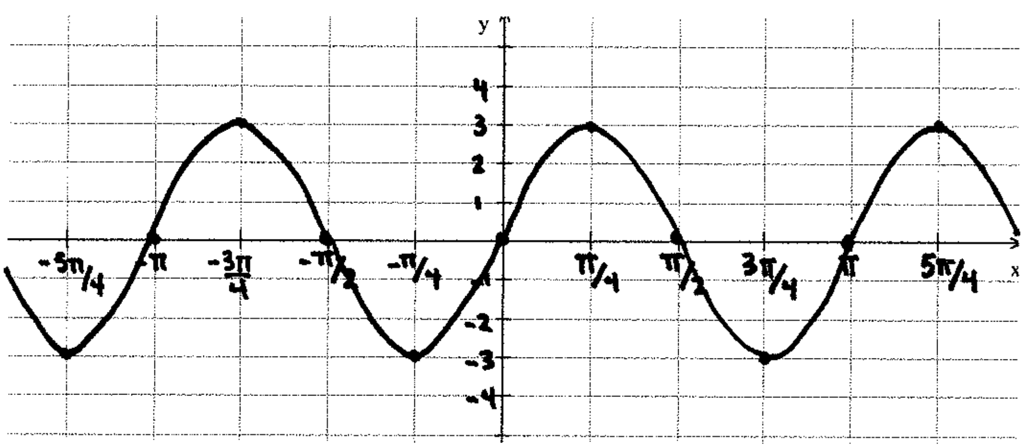


b) $y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$

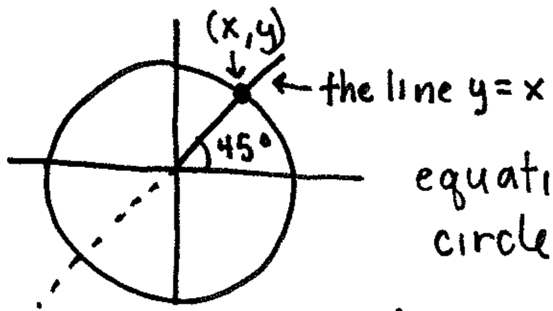
Amplitude = 3

Period = $\frac{2\pi}{2} = \pi$

Phase shift = $\pi/4$ ← this is a right shift



BONUS (3 points): Explain, using a unit circle, why sin 45° and cos 45° are both $\frac{\sqrt{2}}{2}$. You can use diagrams, equations, words, etc., as long as your explanation is easy to follow.



equation for the unit circle: $x^2+y^2=1$

but since the point (x,y) is on the line $y=x$, we know:

$$x^2+x^2=1$$

$$2x^2=1$$

$$x^2=\frac{1}{2}$$

$$x=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$y=x, \text{ so } y=\frac{\sqrt{2}}{2}$$

then since cosine & sine of an angle are given by the coordinates where the terminal side meets the unit circle, $x=\frac{\sqrt{2}}{2}=\cos(45°)$ & $y=\frac{\sqrt{2}}{2}=\sin(45°)$