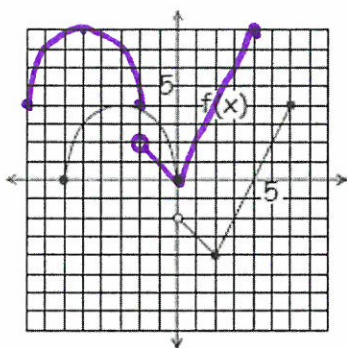


Lab 3: Manipulating Graphs

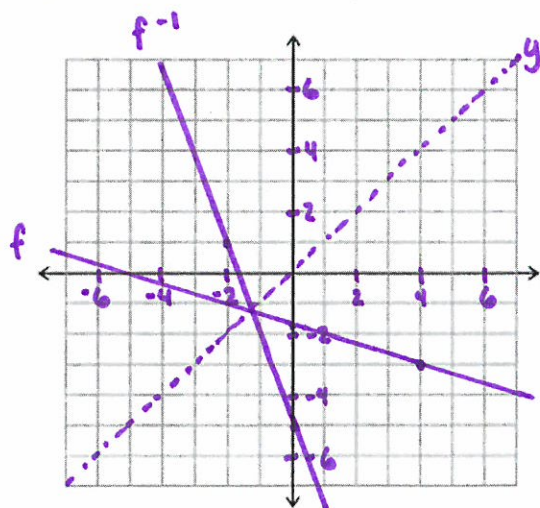
In this lab you will do several problems that will help you explore the ideas of transformation, combining functions, and inverse functions in terms of their effects on graphs. Do the problems neatly in pencil. Clearly lay out your work using proper notation. If you need more space, attach additional paper (if you do so, make sure you number your problems). If a problem involves algebraic work, circle/highlight/box your final answer. This lab is worth 40 points.

Task 1 (6 pts): Given the graph of $f(x)$ below, sketch $f(x+2)+4$ on the same set of axes. Note that you do NOT need to come up with an algebraic expression for $f(x)$.



$f(x+2)+4$
 ↑ shift left 2 units
 ↑ shift up 4 units

Task 2 (10 pts): Find the inverse of $f(x) = \frac{-x-5}{3}$ algebraically, showing your work. Then graph both functions on the axes below, as well as the line $y=x$. Label the scale of your axes. Then verbally explain how your graph tells you that you calculated the inverse correctly.



| x | $f(x)$ | x | $f^{-1}(x)$ |
|-----|--------|-----|-------------|
| 0 | $-5/3$ | 0 | -5 |
| 4 | -3 | -2 | 1 |

$$\rightarrow = -\frac{1}{3}x - \frac{5}{3}$$

$$y = \frac{-x-5}{3}$$

↓

$$x = \frac{-y-5}{3}$$

$$\cdot 3 \quad \cdot 3$$

$$3x = -y-5$$

$$+5 \quad +5$$

$$3x+5 = -y$$

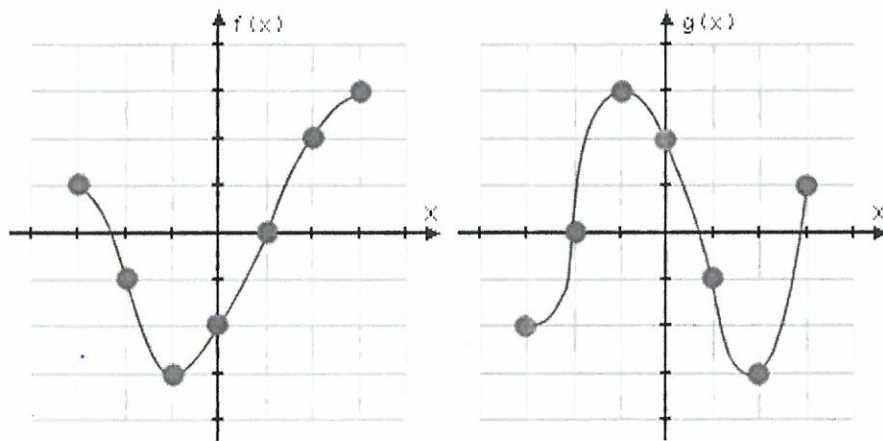
$$\frac{-1}{-1} \quad \frac{-1}{-1}$$

$$-3x-5 = y$$

$$\boxed{f^{-1}(x) = -3x-5}$$

I know that f^{-1} is correct, since it is the reflection of f over the line $y=x$.

Task 3 (6 pts): Using the graphs of $f(x)$ and $g(x)$ below, fill out the rest of the table. Show your work, following my example.



| x | $(g \circ f)(x)$ |
|-----|------------------|
| -3 | -1 |
| -2 | 3 |
| -1 | -2 |
| 0 | 0 |
| 1 | 2 |
| 2 | -3 |
| 3 | 1 |

$$g(f(-3)) = g(1) = -1$$

$$g(f(-2)) = g(-1) = 3$$

$$g(f(-1)) = g(-3) = -2$$

$$g(f(0)) = g(-2) = 0$$

$$g(f(1)) = g(0) = 2$$

$$g(f(2)) = g(2) = -3$$

$$g(f(3)) = g(3) = 1$$

Task 4 (6 pts): Using the graphs of $f(x)$ and $g(x)$ above, calculate each of the following, showing your work. Note that "undefined" is a possible answer.

$$a) (f + g)(2) = f(2) + g(2) = 2 + (-3) = \boxed{-1}$$

$$b) (g - f)(-1) = g(-1) - f(-1) = 3 - (-3) = \boxed{6}$$

$$c) \left(\frac{g}{f}\right)(1) = \frac{g(1)}{f(1)} = \frac{-1}{0} \text{ undefined}$$

$$d) (fg)(3) = f(3) \cdot g(3) = 3 \cdot 1 = \boxed{3}$$

$$e) (g \circ g)(-1) = g(g(-1)) = g(3) = \boxed{1}$$

$$f) (f \circ f)(0) = f(f(0)) = f(-2) = \boxed{-1}$$

Task 5 (12 pts): The following functions are transformations of the graph of $f(x) = x^2$. Write each function in terms of $f(x)$, then verbally explain how to transform $f(x)$ into the given function. Finally, choose the graph that matches the function.

Example: If I gave you $y = 2x^2 + 3$, you would say " $y = 2f(x) + 3$, so you stretch f vertically by a factor of 2 and move it up 3 units".

a) $y = x^2 + 2$ $y = f(x) + 2$, so you shift f up 2 units \rightarrow graph ii

b) $y = (x - 1)^2$ $y = f(x - 1)$, so you shift f right 1 unit \rightarrow graph iv

c) $y = -x^2 + 1$ $y = -f(x) + 1$, so you reflect f over the x -axis & shift it up 1 unit \rightarrow graph i

d) $y = \frac{1}{2}x^2 - 2$ $y = \frac{1}{2}f(x) - 2$, so you shrink f vertically by a factor of $\frac{1}{2}$ & shift it down 2 units \rightarrow graph iii

