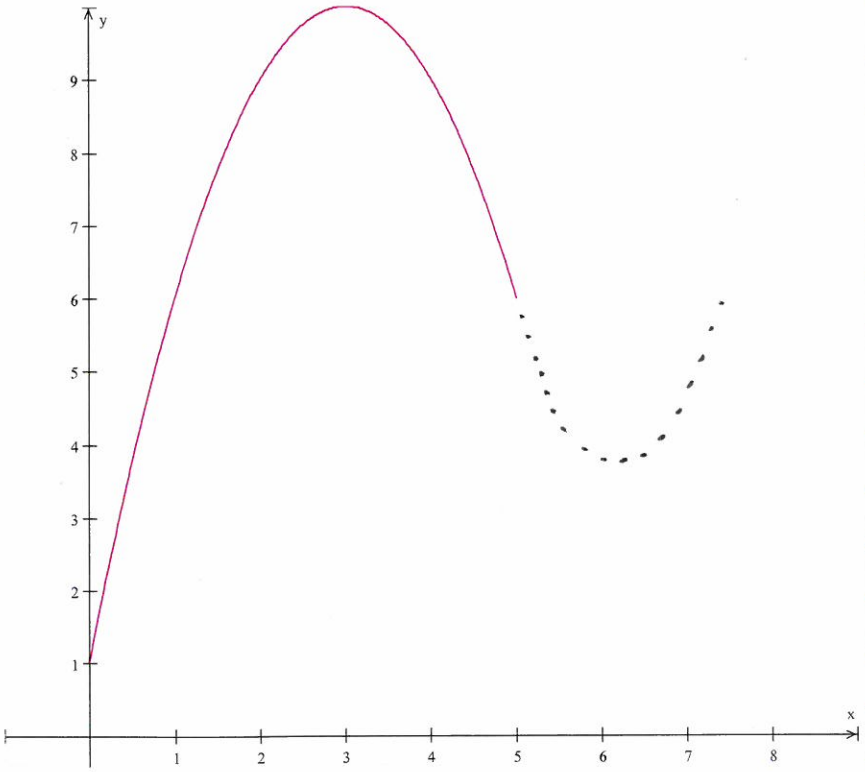
How to Splice Functions: An Example

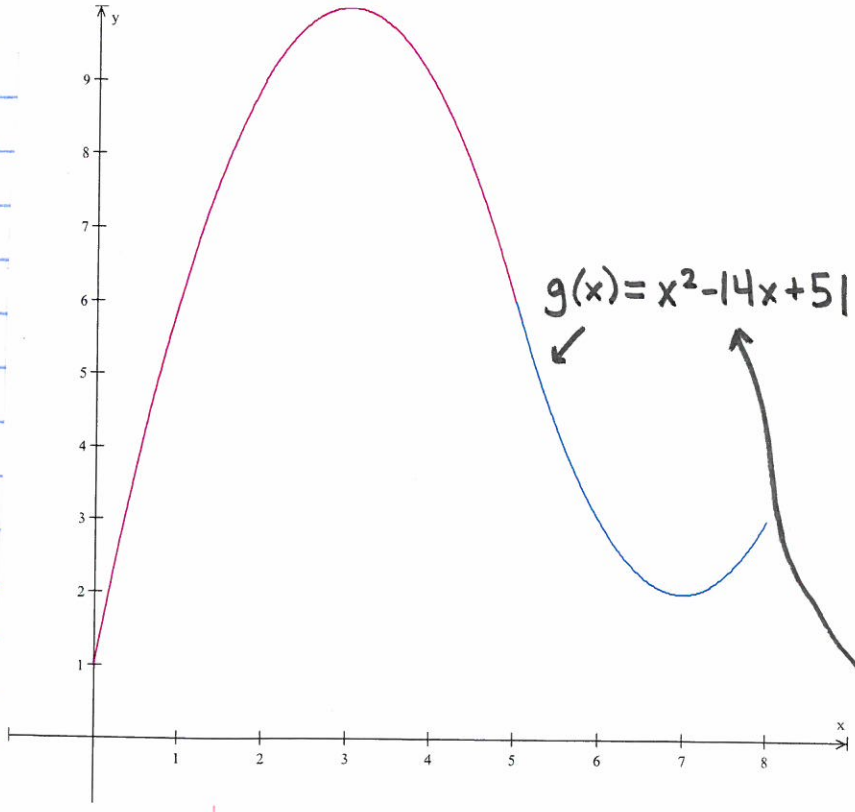


The parabola to the left is the graph of $y = -x^2 + 6x + 1$, defined for $0 \le x \le 5$.
I want to splice an upward-opening parabola to it at $x = 5$ – something like the dotted line.
That means I need its value & derivative to match my value and derivative at $x = 5$.

If $f(x) = -x^2 + 6x + 1$, then $f'(x) = -2x + 6$, so $f(5) = 6$ and $f'(5) = -4$

We want some $g(x) = ax^2 + bx + c$ where $a > 0$, $g(5) = 6$, and $g'(5) = -4$

$g(5) = 6 \rightarrow a \cdot 5^2 + b \cdot 5 + c = 6$
$\quad * \; 25a + 5b + c = 6$
$g'(x) = 2ax + b$ so
$g'(5) = -4 \rightarrow 2a \cdot 5 + b = -4$
$\quad * \; 10a + b = -4$
So we just need a, b, and c that satisfy the starred equations. Let $a = 1$.
Then $10 + b = -4 \rightarrow b = -14$ &
$25 - 5(14) + c = 6 \rightarrow c = 51$



But what if I have stricter requirements for $g(x)$? For example, I may want its vertex to be at $x=6$ rather than $x=7$ as in the previous example.

In that case, I want $-b/2a=6$.
But I also still need $25a+5b+c=6$ and $10a+b=-4$
So I use substitution: $-b=6\cdot 2a$

$$b=-12a$$

$$10a+(-12a)=-4 \rightarrow -2a=-4 \rightarrow a=2$$
$$b=-12(2)=-24$$
$$25(2)+5(-24)+c=6 \rightarrow -70+c=6 \rightarrow c=76$$

So $g(x)=2x^2-24x+76$ also splices in, as shown below.