Lab #6 Name(s)____

Roller Coaster Prep. Show all work to derive the following equations. Pencil only (or erasable pen).

(45 points)

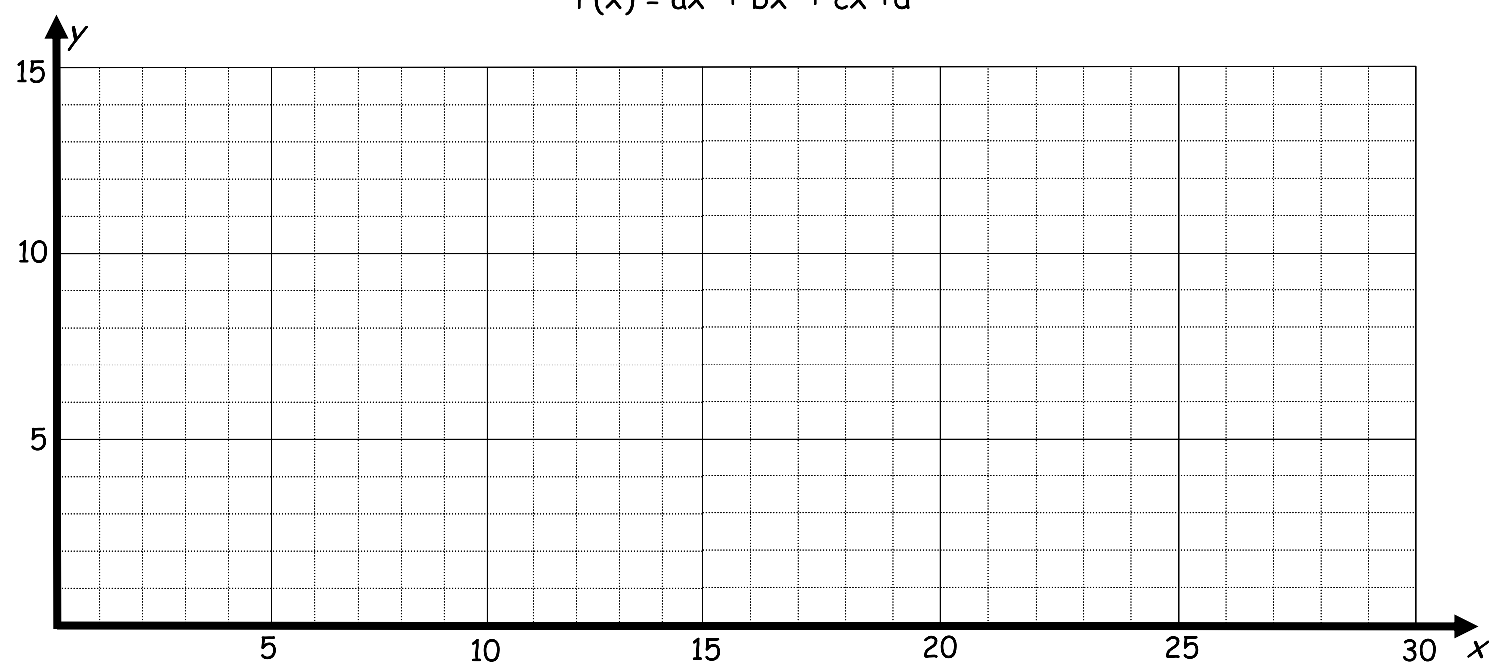
The roller coaster design is built by defining the entire curve piece-wise; that means connecting a series of functions together. We'll scale down by a factor of 10 here. Recall that your roller coaster takes off from an elevation of zero with a slope of a tangent line of zero. We will build two different cubic equations where the maximum of the first is at (10, 15), minimum is at (0, 0), and the second is attached to the first at a "nice" point. Also recall that the max height of the coaster is 150 ft (thus y = 15) and x = 10 was arbitrarily picked since it was a nice number.

First question: What is the slope of the tangent line to a cubic at its local maximum or minimum?

Slope is______________, so $P'(x_{max/min})$ = ________

Just plot the points (0,0) and (10,15). These will be the local minimum and maximum of a cubic function of the form:

$$P(x) = ax^3 + bx^2 + cx + d$$



Our goal here is to use the information just given to find a, b, c, and d, the parameters of the cubic equation. Write the following in terms of the parameters and state what they are each equal to:

$P(0)$ =

$P(10)$ =

$P'(x)$ =

$P'(0)$ =

$P'(10)$ =

At this point, you should be able to generate two equations and two unknowns, use them to find the missing parameters and write the equation of the cubic here:

Find P(13) =_________ ; P'(13) = _________ and represents?_____________________________

We're going to use (13, P(13)) as our "nice" point to anchor our next cubic to. Use the TABLE feature on your calculator to carefully plot points for each integer value of x in the interval $x \in [0,13]$. Now sketch the graph of P. We'll label it P_1 as our first polynomial.

Before you go any further, did you get $P_1(x) = \frac{-3}{100}x^3 + \frac{9}{20}x^2$? Double-check your work. It needs to support your answer. Next, our goal is to "connect" another section of a cubic function, temporarily call it Q, which passes through (13, 10.14) with a tangent slope of -3.51 at x = 13.

Recall from the roller coaster project that the next height can't be more than 75% the previous: 75% of 15 is 11.25, I chose 10.14 (it ended up giving me some "nicer" numbers, as you'll see). I also chose the height y = 10.14 to occur at x = 23.14 (temporarily at x = 10.14). To do this we'll "build" the next section starting at x = 0 and then do a horizontal shift of 13 units so our new function will look like a continuation of the old function. The new conditions that must be satisfied by Q:

Q(0) = 10.14	Q(10.14) = 10.14
Q'(0) = -3.51	Q'(10.14) = 0

Use this information, which generates 4 equations and 4 unknowns, to find a cubic equation with coefficients that can be expressed as fractions or, if possible, an exact decimal to 2 decimal places.

Using $Q(x) = ax^3 + bx^2 + cx + d$, write the 4 equations generated in terms of the parameters. It will be helpful to leave powers of 10.14 as powers.

Q(0) =

Q(10.14) =

Q'(0) =

Q'(10.14) =

The function you derived: Q(x) = ______________________________

The graph of the second equation now has to be shifted to the right 13 units by creating a new function using the following, each "x" in Q gets replaced with (x-13):

$P_2(x) = Q(x-13) =$

Now graph P_2 by using the TABLE feature on your calculator to carefully plot points for each integer value of x in the interval $x \in [13,30]$. Is there a "nice" point in the interval to attach a third cubic function? What point? What's its tangent line's slope?