Calculus Lab #5 Name(s):____
More Fun with Derivatives (85 pts).

In this lab we will work with derivatives to gain an even better understanding of them. As usual completeness, neatness and accuracy are all weighed evenly when calculating your score. Please write your answers only here and show all necessary work on a separate paper. 10% added back for working in groups and submitting one report.

1. Here near the Earth's surface we can approximate the position of a free-falling object by $s(t) = -16t^2 + v_o t + s_o$, where s(t) is in feet and t is in seconds, v_o is the initial velocity and s_o is the initial position. It turns out that a rocket was fired from the edge of 100 foot cliff with an initial velocity of 400 ft/sec. (15 points)

   a) How high did the rocket go?

   b) How long did it take it to reach the highest point.

   c) Find s(4) and s'(4). What do these numbers represent in the context of this rocket problem?

   d) How long does it take the rocket to reach the ground? (Below the cliff, round to nearest 100^{th})

   e) How fast is the rocket going as it hits the ground?

2. To the nearest 100^{th}, for what values of x does $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent line? Show work. Recall, set the derivative equal to zero and solve. Why do we set $f'(x) = 0$? (5 pts)

3. Find equations of both lines that are tangent to the curve $y = 1 + x^3$ and are parallel to the line $12x – y = 1$. Draw a sketch of the graph of the situation. It may be helpful to let the units on the x-axis be one box length (one unit) and the units on the y-axis where one box length is 2 or 4 units, I have graph paper if you need some. (10 pts)

4. (10 pts) Let $f(x) = x^x$.

a) To four decimal accuracy, use the limit definition, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$ of the derivative to approximate f'(2) - the answer is not 4! - by letting h get smaller and smaller. On your paper show for at least 5 values of h.

a) f'(2) =

b) Also, find $\lim_{x\to 0^+} f(x) =$

5. (5 pts) Find the equation of the tangent line of y = f(x) at x = 2 if f(2) = -3 and f '(2) = ½.

6. (5 pts) The tangent line of y = g(x) at the point (2, -3) travels through the point (-1, -5).

   Find g(2) =

   and g'(2) =

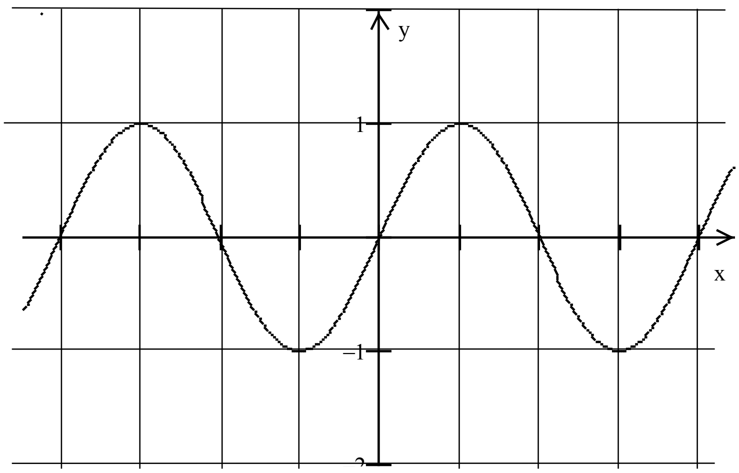
7. (10 pts) Let f(2) = 3, f '(2) = -1, g(2) = -4, and g '(2) = 5 (Section 3.2 is needed for this problem)

   a) If h(x) = f(x)g(x), find h '(2) =

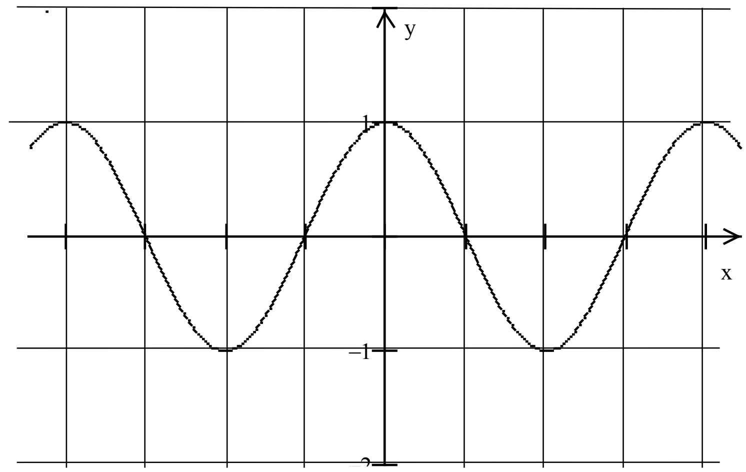
   b) If k(x) = f(x)/g(x), find k '(2) =

8. (10 pts) Assume the following two graphs are graphs of two very basic trig functions; identify and label the graphs represented by each. What is the length of each of their cycles (called the period = P)?
What are the units on each x-axis? y-axis?
Draw the derivative of each function. (Hint, where are the slopes of the tangent lines equal to zero? Can you identify the graph of each derivative?

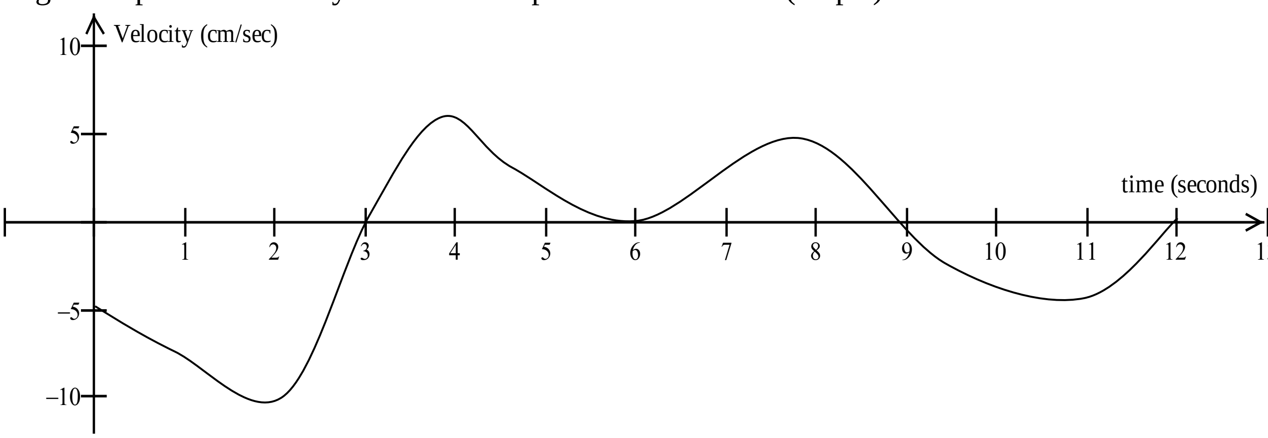
a)



b)



9. Another hungry mouse is in the middle of a long tube at time = 0. Nobody is sure how he got there, but when t = 0 he is in the middle of the tube. He runs back and forth, as some smelly food is placed then removed from each end of the tube. This is not hurting the mouse in any way. Think of it as mouse exercise. Below is the graph of the mouse's VELOCITY in cm/sec as a function of time. Assume movement to the right is a positive velocity. Be sure to explain each answer. (15 pts)



a) Which way did the mouse run first? Was the mouse tossed into the tube or placed.

b) When was the mouse moving the fastest? What direction was he going?

c) At what time(s) did the mouse turn around?

d) On what intervals is the mouse speeding up? Slowing down?

e) On what intervals was the mouse moving to the right?

f) Describe what is happening near t = 6.

g) Relative to the center of the tube, where do you think the mouse is at t = 12? Why?