Calculus Lab #4 **Name(s):**____

Rates of Change

The idea of this lab is to continue studying graphs as well as **rates of change**. A rate of change examines how one value changes with respect to another – think of velocity, or speed of an object, as the rate of change of how distance changes with respect to time: miles *per* hour, feet *per* second, etc. A rate compares one number with another in the form of a fraction and *per* means to divide. As usual, completeness, neatness and accuracy are all weighed evenly when calculating your score. Round to 2 decimals unless otherwise specified.

Write only your answers here and show all necessary work on a separate paper.

1. Here near the Earth's surface we can approximate the position of a free-falling object by

$$s(t) = -16t^2 + v_o t + s_o,$$

where s(t) is in feet and t is in seconds, v_o is the initial velocity and s_o is the initial position.

a) Sam threw a ball straight up where $v_o = 128$ and $s_o = 4$. Find the average velocity from t = 0 to t = 3.

b) With the same given information as part a), find the instantaneous velocity at t = 3 by finding the following limit: $\lim_{t \to 3^-} \frac{s(t)-s(3)}{t-3} =$

c) What direction was the ball going at t = 3?

d) How long will it take to hit the ground, rounding to 2 decimals?

e) What is the ball's *velocity* when it hits the ground?

2. Find the slope of the tangent line at x = 0 for the following functions to **4 decimal accuracy using****:

a) $f(x) = 2^x$, slope is approximately:

and b) $g(x) = 3^x$ slope is approximately:

**Recall that this is done by finding the following limit shown below. The problem is that we don't have the algebra "trick" to get rid of the h. So in this case let h be a real small number such as h = 0.001. This of course is not the same as letting $h \to 0$, but at this point it is a good approximation.

$$** \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

(The x is replaced by 0, because we want the slope at x = 0)

Show the values you got as you let h get smaller on your separate paper.

At a minimum, use the values of h given in the following table for #3 to answer #2.

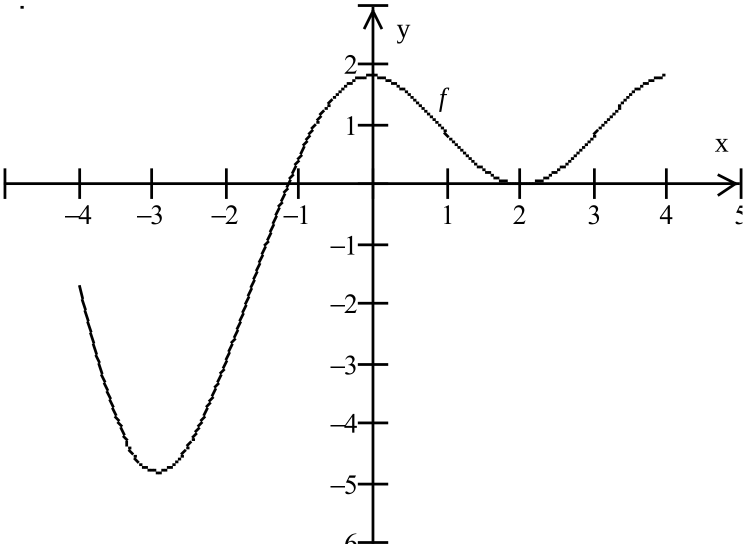
3. Of course, I am going to assume that you did #2 correctly. If so, you found that the slope of $f(x) = 2^x$ at x = 0 is less than one, and the slope of $g(x) = 3^x$ at x = 0 is more than one. It seems reasonable, that there must be a number, "c" that is between 2 and 3 where the slope of $f(x) = c^x$ at x = 0 is exactly one. In fact this number, c, is actually our friend "e".

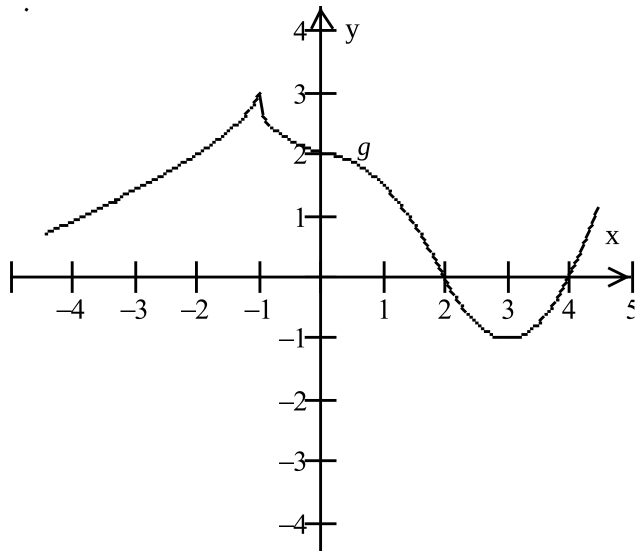
Follow the same directions as with #2 by filling in the table, but this time let $k(x) = e^x$.

For h=	**.1**	**.001**	**.0001**	**.00001**	**.000001**	**h→0**
$\frac{k(0+h)-k(0)}{h} =$						

Record to 4 decimal places for each entry in the table.

4. Carefully sketch each graph's derivative on the associated graph and label your new graphs appropriately:



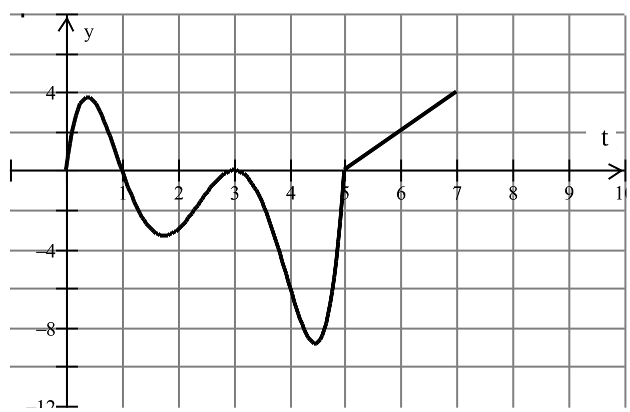


5. a) Find the equations of the tangent lines of $f(x) = x^2 + 2x - 3$ where f(x) crosses the x-axis.
Equations are:

b) Sketch a graph of f(x) and the two tangent lines from part a onto your work paper.

c) Find the point of intersection of the two tangent lines. Point of intersection is:

6. A hungry mouse is in the middle of a long tube at time t = 0. Nobody is sure how he got there, but when t = 0 he is in the middle of the tube. He runs back and forth as some smelly food is placed then removed from each end of the tube.



This is not hurting the mouse in any way. Think of it as mouse exercise. To the right is the graph of the mouse's VELOCITY in cm/sec as a function of time. **Assume movement to the right is a positive velocity, to the left is negative. Be sure to explain each answer.**

a) Which way did the mouse run first? Was the mouse thrown into the tube or placed.

b) When was the mouse moving the fastest? What direction was he going?

c) When did the mouse turn around?

d) On what intervals is the mouse speeding up? Slowing down?

e) On what intervals was the mouse moving to the right?

f) Describe what is happening near t = 3.

g) What is happening on the interval (7, 9)

h) Relative to the center of the tube, where is the mouse at t = 5?