Calculus Lab #3 Name(s):____
An Introduction to Limits
Show all your work on a separate paper and write your answers on this sheet in pencil only, please.

The idea of this lab is to further your understanding of the idea of a limit. As usual; completeness, neatness and accuracy are all considered when calculating your score. (75 points total at 5 points each).

To begin with, I'll introduce a little notation. $\lim_{x \to a} f(x) = L$.

This says, "The limit of f(x), as x approaches a, equals L". In other words, when x gets really close to "a", y gets really close to L.

Note that we are not saying f(a) = L. Often f(a) does equal L, but sometimes f(a) is actually undefined, but the limit is not. To find a limit you can substitute values for x that get closer and closer to "a", or you can look at the graph of y = f(x) to see what value y gets close to as x gets close to "a".

Find the following limits (show work on a separate paper neatly, it will be turned in with this lab):

1. $\lim_{x \to 2} \dfrac{x - 2}{x^2 - 4} =$

2. $\lim_{x \to -3} \dfrac{x^2 - 9}{x + 3} =$

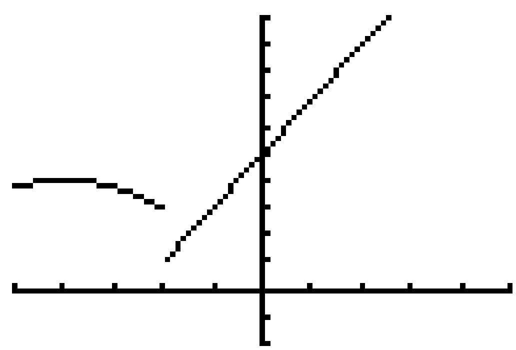
3. $\lim_{x \to 0} \dfrac{x}{\sqrt{x + 16} - 4} =$

We often look at limits from one side. We might say, what happens to f(x) if x approaches "a" from the left. This means that x is always smaller than "a". This one sided limit is written: $\lim_{x \to a^-} f(x) = L$ (Note the "-" sign above the "a") This is often referred to as a left-hand limit. There is also a right-hand limit. For the right-hand limit, x is always bigger (or to the right of "a"). This limit is written: $\lim_{x \to a^+} f(x)$. (Note the "+" above the "a").

4. Use the graph given below to approximate the following limits:

a) $\lim_{x \to -2^-} f(x) =$

b) $\lim_{x \to -2^+} f(x) =$



Sometimes a limit simply does not exist. In fact we say that a limit is L, if and only if the two one sided limits both equal L. In symbols, it looks like this:

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L$$

5. Based on this discussion does $\lim_{x \to -2} f(x)$ exist? Why or why not?

Sometimes a limit does not exist, but we can still describe what is happening. This occurs when the value of f(x) becomes an arbitrarily large number or arbitrarily large negative number. In these cases we say the limit is infinity or negative infinity. It looks like this:

$\lim_{x\to a} f(x) = \infty$ or $\lim_{x\to a} f(x) = -\infty$. Keep in mind, we are not saying that ∞ is a number or that the limit exist. In the first case we are simply stating that as x gets close to a, f(x) grows arbitrarily large. A similar explanation goes with the case where the limit equals -∞.

For number 6 and 7 find the following limits:

6. $\lim_{x\to 2^+} \frac{4x}{x - 2} =$

7. $\lim_{x\to 2^-} \frac{4x}{x - 2} =$

8. $\lim_{x\to 2} \frac{4x}{x - 2} =$

9. $\lim_{x\to 0^+} \ln(x) =$

Often we are concerned with what happens to f(x) as x goes to infinity or negative infinity. The limits look like: $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$. If these limits exist, they are often the horizontal asymptotes of f(x).

Find the following limits:

10. $\lim_{x\to\infty} \frac{3x}{4x - 8} =$

11. $\lim_{x\to\infty} \frac{4x}{x^2 - 2x} =$

All of the previous limits we can find by looking at graphs or substituting values for x that get really close to "a". We also need to understand the algebra of limits. That is, we need to do some algebraic manipulations first, then evaluate the limit. We have seen some of this in class already in the form of the difference quotient. For example if $f(x) = x^2 - 2x + 3$

$$\lim_{h\to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h\to 0} \frac{(x + h)^2 - 2(x + h) + 3 - (x^2 - 2x + 3)}{h} =$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h + 3 - x^2 + 2x - 3}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h} = \lim_{h\to 0} \frac{h(2x + h - 2)}{h} = \lim_{h\to 0} (2x + h - 2) = 2x - 2$$

Find the limit of the difference quotient, $\lim_{h\to 0} \frac{f(x + h) - f(x)}{h}$, for the following functions:

12. $f(x) = 5x - 3$

13. $f(x) = x^2 + 4x - 5$

14. $f(x) = 3x^2 + 4x - 5$

15. $f(x) = x^3 - 2x^2 + 3x - 2$