Math 253 Lab #7 Power Series, an Intro. Names: ______________________________

Work in groups of 2-4 for full credit and 5% bonus if turned in by Wed. of week 10. Make sure everyone in your group understands the question and its answer. Work neatly and in pencil, with any additional work attached to this lab. Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. **Remember that you are practicing your <u>mathematical communication</u> skills!** Labs turned in after Wed. are reduced 25%, only accepted through Friday of week 10.

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Part 1: The Geometric Series with a Variable.

Let's do something different with geometric series. Instead of just working with numbers, let's include a variable and see what happens:

<u>example:</u> $\sum_{n=0}^{\infty}(x-12)^n = 1+(x-12)+(x-12)^2+(x-12)^3+\cdots = ?$

<u>step 1: Is it a geometric series? If it is, what's r?</u>

This is a geometric series, and r = x - 12.

<u>step 2: For what interval will the series converge?</u>

The series will converge iff $|r|<1$, so...

$|x-12|<1$

split the absolute value into positive and negative

$-1<x-12<1$

add 12 to all three sections of the inequality

$11<x<13$

<u>step 3: Find the sum of the geometric series. If necessary, factor to force the first term to be one, then use the formula.</u>

$$\sum_{n=0}^{\infty}(x-12)^n = 1+(x-12)+(x-12)^2+(x-12)^3+\cdots$$

the first term is already 1, so don't factor

$$=\frac{1}{1-r}$$

$$=\frac{1}{1-(x-12)}$$

$$=\frac{1}{13-x}$$

<u>step 4: Summarize your results.</u>

$$\boxed{\sum_{n=0}^{\infty}(x-12)^n = \frac{1}{13-x} \quad \text{if and only if} \quad 11<x<13}$$

<u>step 5: Check your answer by graphing on your calculator.</u>

$y_1 = 1/(13-x)$

$y_2 = 1+(x-12)+(x-12)^2+(x-12)^3+\cdots$ plus as many terms as you want...

Do the graphs line up between x = 11 and x = 13?

Now try these problems, showing similar steps.

For each problem, start by writing out the first few terms of the series. Remember to check your answer by graphing on the calculator, and remember to summarize your results at the end of each question.

(1) $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$

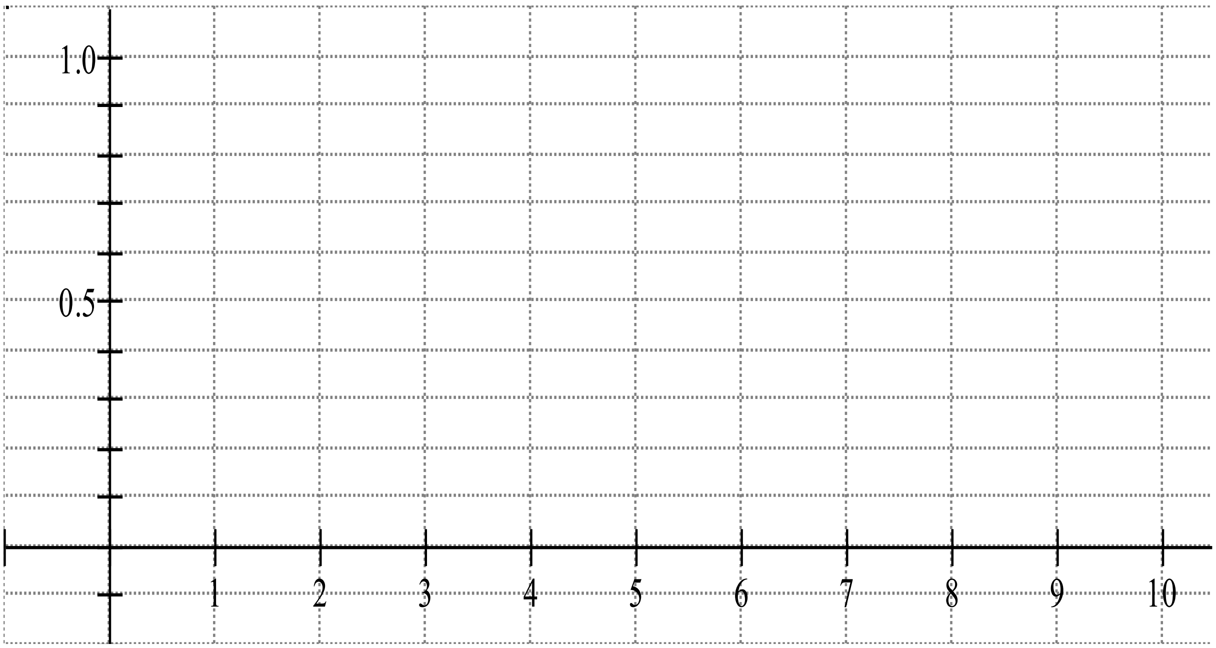
(2) $\sum_{n=0}^{\infty}5x(x-2)^n$

(3) $\sum_{n=1}^{\infty}(3x)^n$

Part 2: Deceptive Visions
(Based on Instructor's Manual, Stewart Calc., 4th ed.)

(1) Graph the following three functions on the same set of axes. Use 3 different colors, or 3 different line patterns to distinguish the graphs. <u>Label them clearly! You have to show me that you know which one is which.</u>

$$f(x)=\frac{1}{x^{0.99}} \quad g(x)=\frac{1}{x^{1}} \quad h(x)=\frac{1}{x^{1.01}}$$



(2) Can you see a significant difference among the three graphs? If so, what is it; if not, why not?

(3) Using your calculator, compute the areas under the three curves between x = 1 and x = t. Round to 3 decimal places.

	$\int_1^t \frac{1}{x^{0.99}}\,dx$	$\int_1^t \frac{1}{x^{1}}\,dx$	$\int_1^t \frac{1}{x^{1.01}}\,dx$
t = 10			
t = 50			
t = 100			

(4) What do you notice about the numbers on the table? Do the three functions have similar areas? Do you see any important differences?

As you go further down the table, the numbers get ____________________, *because*

____

As you go to the right on the table, the numbers get ____________________, *because*

____

(5) What do you think will happen to the area under each of the three functions if you let t go to infinity? Try it -- showing your work, solve each of these improper integrals:

(a) $\int_1^\infty \frac{1}{x^{0.99}}\,dx$

(b) $\int_1^\infty \frac{1}{x}\,dx$

(c) $\int_1^\infty \frac{1}{x^{1.01}}\,dx$

(6) Summarize your results: An integral of the form $\int_1^\infty \frac{1}{x^p}\,dx$ will converge if and only if

____. It will diverge if ________________________.

Part 3: Practice with The Alternating Harmonic Series.

(1) Find the first few partial sums of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+\ldots$. (Show your answers as <u>decimals</u>, but don't round them off! Keep all the digits your calculator gives you.) This series is called the <u>alternating harmonic series</u>.

$$1=$$

$$1-\frac{1}{2}=$$

$$1-\frac{1}{2}+\frac{1}{3}=$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}=$$

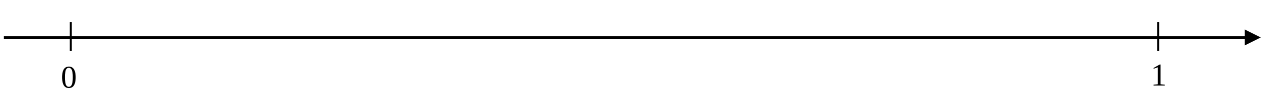
$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}=$$

(2) Find the sums of the first 498 through 502 terms of the alternating harmonic series. (Remember the calculator syntax: sum(seq((-1)^(n+1)/n,n,1,#))
where # is the total number of terms you want to add.)

498 terms	
499 terms	
500 terms	
501 terms	
502 terms	

(3) How many decimals of accuracy do you think your answer has at this point? (To what place value do <u>all</u> the answers agree?)

(4) Will the alternating harmonic series converge? <u>Explain</u> your answer, being very specific. You may find it helpful to draw a diagram of some sort.



Part 4: The Harmonic Series adds up VERY slowly.

We've proved in class that the harmonic series adds up to infinity:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \ldots\ldots\ldots\ldots = \infty$$

Let's explore just how slowly it adds up. For each total given below, how many terms of the harmonic series do you need to reach that total?

Total Sum	Number of Terms needed.
1	1
2	4
3	11
4	
5	
6	
7	
8	
9	

On your TI-83/84 calculators, you can add at most 999 terms at a time using "sum(seq(blah,n,a,b))", so try this program instead:

```
Prgm: HARMONIC
:0 → S
:Disp "HOW MANY TERMS?"
:Prompt T
:For (N,1,T)
:S+1/N → S
:Disp S
:End
:Stop
```

→ is the STO key

So … if you tried to add this up by hand, and you could add one more fraction to the sum every minute, and you worked all day every day adding fractions, how long (how many days, hours, and minutes) would it take you to get the total to be 9?