Math 253 Lab #6 Sequences and Series, an Intro. Names: ______________________

Work in groups of 2-4 for full credit and 5% bonus if ______________________
turned in by Wed. of week 9. Make sure everyone in your ______________________
group understands the question and its answer. Work ______________________
neatly and in pencil, with any additional work attached to this lab. Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. **Remember that you are practicing your <u>mathematical communication</u> skills!** Labs turned in after Wed. are reduced 25%, only accepted through week 9. WR HW 8 is an addendum to this lab; everyone turns in their own HW.

Part 1: Practice with Convergence of Sequences.

Do the following sequences converge or diverge? If a sequence converges, specify what it converges to. Justify your answers -- write the first 5 terms of the sequence (start with n=1), and then <u>explain</u>. (Remember, "explain" means use words, put together in complete sentences!)

Write the terms as **<u>decimals</u>**, with at least 4 digits.

(1) $a_n = \left(\frac{e}{3}\right)^n$

This sequence converges/diverges to:

$a_1 =$

$a_2 =$

because:

$a_3 =$

$a_4 =$

$a_5 =$

(2) $a_n = (-1)^n \sqrt{n}$

This sequence converges/diverges to:

because:

(3) $a_n = (-1)^n \frac{1}{\sqrt{n}}$

This sequence converges/diverges to:

because:

(4) $a_n = (-1)^{2n+1}$

This sequence converges/diverges to:

because:

Part 2: Practice with Finite Series.

- Write out the individual terms of each sum, up through n = 4.
- Find the total sums, at least up through n = 6.
- Find a formula that expresses the total sum in terms of n.

(1) $\sum_{i=0}^{n} 2^i$

Hint:

$$\sum_{i=0}^{0} 2^i = 2^0 = 1 \qquad = 2^1 - 1$$

$$\sum_{i=0}^{1} 2^i = 2^0 + 2^1 = 1 + 2 = 3 \qquad = 2^2 - 1$$

$$\sum_{i=0}^{2} 2^i = 1 + 2 + 4 = 7 \qquad = 2^3 - 1$$

$$\sum_{i=0}^{3} 2^i =$$

$$\sum_{i=0}^{4} 2^i =$$

$$\sum_{i=0}^{5} 2^i =$$

$$\sum_{i=0}^{6} 2^i =$$

$$\vdots$$

$$\sum_{i=0}^{n} 2^i =$$

(2) $\sum_{i=0}^{n} 3^i$

Think about the previous problem and what you might have to divide by to obtain each of the sums:

$$\sum_{i=0}^{0} 3^i = 3^0 = 1$$

$$\sum_{i=0}^{1} 3^i = 3^0 + 3^1 = 1 + 3 = 4$$

$$\sum_{i=0}^{2} 3^i =$$

$$\sum_{i=0}^{3} 3^i =$$

$$\sum_{i=0}^{4} 3^i =$$

$$\sum_{i=0}^{5} 3^i =$$

$$\sum_{i=0}^{6} 3^i =$$

$$\vdots$$

$$\sum_{i=0}^{n} 3^i =$$

(3) $\sum_{i=0}^{n} 4^i$

$$\sum_{i=0}^{0} 4^i = 4^0 = 1$$

$$\sum_{i=0}^{1} 4^i = 4^0 + 4^1 = 1 + 4 = 5$$

$$\sum_{i=0}^{2} 4^i =$$

$$\sum_{i=0}^{3} 4^i =$$

$$\sum_{i=0}^{4} 4^i =$$

$$\sum_{i=0}^{5} 4^i =$$

$$\sum_{i=0}^{6} 4^i =$$

$$\vdots$$

$$\sum_{i=0}^{n} 4^i =$$

(4) Find the formula for this one:

$$\sum_{i=0}^{n} r^i =$$

Part 3: More Practice with Convergence of Sequences.

Do the following sequences converge or diverge? If a sequence converges, specify what it converges to. Justify your answers -- write the first 5 terms of the sequence (start with n=1), and then explain. (Remember, "explain" means use words, put together in complete sentences!)

Write the terms as decimals, with at least 4 digits.

(1) $a_n = \dfrac{(-1)^n + n}{(-1)^n - n}$

1st 5 terms:

This sequence converges to:

because:

(2) $a_n = \dfrac{\ln\left((e^4)^n\right)}{3n}$

1st 5 terms:

This sequence converges to:

because:

(3) $a_n = (-1)^n \cos\left(\dfrac{\pi}{2}(n+1)\right)$

1st 5 terms:

This sequence converges to:

because:

(4) $a_n = (-1)^n \sin\left(\dfrac{\pi}{2}(2n+1)\right)$

1st 5 terms:

This sequence converges to:

because: