Math 253 Lab #5 More on Linear Transformations Names: ______________________

Work in groups of 2-4 for full credit and 5% bonus if turned in by Fri. of week 7. Make sure everyone in your group understands the question and its answer. Work neatly and in pencil, with any additional work attached to this lab. Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. **Remember that you are practicing your <u>mathematical communication</u> skills!** Labs turned in after Fri. are reduced 25% and only accepted through week 8.

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Part 1: Most functions are NOT linear transformations.

A key point while we're learning about linear transformations is remembering that almost every function we've ever dealt with is NOT a linear transformation -- these properties are very unusual. Let's investigate:

Property 1: $f(a+b)=f(a)+f(b)$
Property 2: $f(k\cdot a)=k\cdot f(a) \quad k\in\mathbb{R}$

(1) Let f(x) = sin(x): Get decimals. Calculator in Radians Mode.

Property 1?: $\sin(\frac{\pi}{3}+\frac{\pi}{4})=$

$\sin(\frac{\pi}{3})+\sin(\frac{\pi}{4})=$

are they equal?

Property 2?: $\sin(5\cdot\frac{\pi}{3})=$

$5\cdot\sin(\frac{\pi}{3})=$

are they equal?

(2) Let $f(x)=x^2$: Simplify as much as possible.

Property 1?: $f(a+b)=$

$f(a)+f(b)=$

are they equal?

Property 2?: $f(5a)=$

$5\cdot f(a)=$

are they equal?

(3) Let $f(x)=2x+3$: Simplify as much as possible.

Property 1?: $f(a+b)=$

$f(a)+f(b)=$

are they equal?

Property 2?: $f(5a)=$

$5\cdot f(a)=$

are they equal?

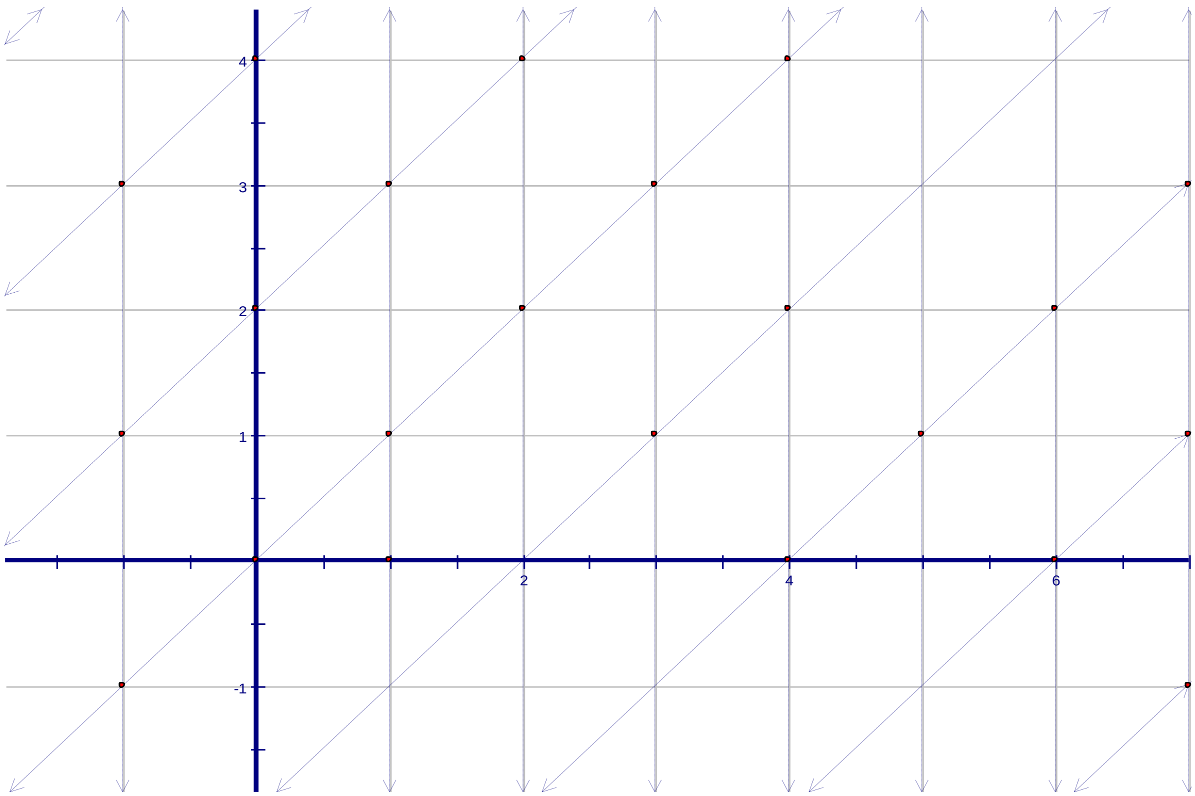
(4) In fact, can you think of ANY function on the real numbers that satisfies the linear transformation properties? There is exactly one type -- what is it?

Part 2: The advantage of linear transformations -- if we know what happens to $\hat{i}$ and $\hat{j}$, we know what happens to <u>every</u> vector in $\mathbb{R}^2$.

Given the matrix $F = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$**, find:** $F\hat{i} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} =$ $F\hat{j} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} =$

$F\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} =$ $F\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} =$ $F\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} =$

Graph all of the above answers on the graph below, labeling each one clearly – use colored pencils and the labels $F\hat{i}$, $F\hat{j}$, $F\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $F\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, and $F\begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Ignore the dotted line grid at first.



Now pay attention to the dotted line grids: Notice that on this graph, I've drawn a grid defined by $F\hat{i}$ and $F\hat{j}$.

(1) Express $F\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ as a linear combination of $F\hat{i}$ and $F\hat{j}$ -- how many $F\hat{i}$'s and $F\hat{j}$'s does it take to create $F\begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

$$F\begin{bmatrix} 2 \\ 1 \end{bmatrix} = F(2\hat{i} + \hat{j}) =$$

(2) Do the same for $F\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

$$F\begin{bmatrix} -1 \\ 2 \end{bmatrix} = F(-1\hat{i} + 2\hat{j}) =$$

(3) Do the same for $F\begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

$$F\begin{bmatrix} 3 \\ -2 \end{bmatrix} = F(3\hat{i} - 2\hat{j}) =$$

So what? In a linear transformation, if we know what happens to $\hat{i}$ and $\hat{j}$, that's enough to tell us what happens to every vector in a linear transformation.

Part 3: Practice solving systems of equations.

Solve each system of equations, finding rref on your calculator, then copying it down. If there is no solution, say so. If there are infinite solutions, describe the solution space (the set of all possible solutions) in vector form. Also, if there are infinite solutions, graph the solution set, on the attached graph paper, back side of this page.

system of equations	**rref**	**set of <u>all</u> solutions**
(1) $x + 2y = 7$ $2x - 4y = 10$		
(2) $x + 2y = 7$ $-2x - 4y = -14$		
(3) $x + 2y = 7$ $x + y = -2$		
(4) $x + 2y = 7$ $x + y = -2$ $2x + 3y = 5$		
(5) $x + 2y = 7$ $x + y = -2$ $2x + 3y = 4$		
(6) $x + 2y = 7$ $x + y = -2$ $4x + 7y = 19$ $3x + 6y = 21$ $-2x - 4y = -14$ ***<u>hint:</u> see p. 62 of your coursepack!***		

Label your graphs. You can use the problem number from the previous page to label your graphs.