

Work in groups of 2-4 for full credit and 5% bonus if turned in by Fri of week 6.

Make sure everyone in your group understands the question and its answer.

Work neatly and in pencil, with any additional work attached to this lab.

Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. I will accept one lab this quarter as an individual project without penalty. Labs turned in after Fri. are reduced by 25%, only accepted through week 7.

Part 1: rref by hand!

Choose one of the following systems of equations, and solve by finding the reduced row echelon form (rref) by hand. Label every step.

Use your own paper.

$$x \quad -3z = -2$$

$$(a) \quad 3x \quad +y \quad -2z = 5$$

$$2x \quad +2y \quad +z = 4$$

$$x \quad +y \quad +z = 6$$

$$(b) \quad x \quad -y \quad +z = 2$$

$$2x \quad +3y \quad +z = 11$$

$$2x \quad +y \quad -z \quad +2w = -6$$

$$(c) \quad 3x \quad +4y \quad \quad +w = 1$$

$$x \quad +5y \quad +2z \quad +6w = -3$$

$$5x \quad +2y \quad -z \quad -w = 3$$

Part 2: A little fun with linear transformations.

Directions for each new matrix (each matrix gets a separate page in this lab):

- Graph and label these four vectors, on the left-hand graph, which will represent a domain. Draw each vector in standard position -- with its tail at the origin. Use a different color for each vector.

$$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \quad \vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

- Then, multiply the given matrix by each vector, and graph and label the resulting vectors on the right-hand graph in the same colors. This right hand graph will represent a range. Again, draw each vector in standard position.

I.e., \vec{t} and $A\vec{t}$ are drawn with the same color, \vec{t} on the left, on the domain graph, and $A\vec{t}$ on the right, on the range graph. Then, \vec{u} and $A\vec{u}$ are drawn with another color, and so on ...

- Then, describe (in words!) what each matrix seems to be doing to the vectors.

Double Check: Label every vector with its name!

$$(1) \quad A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$$

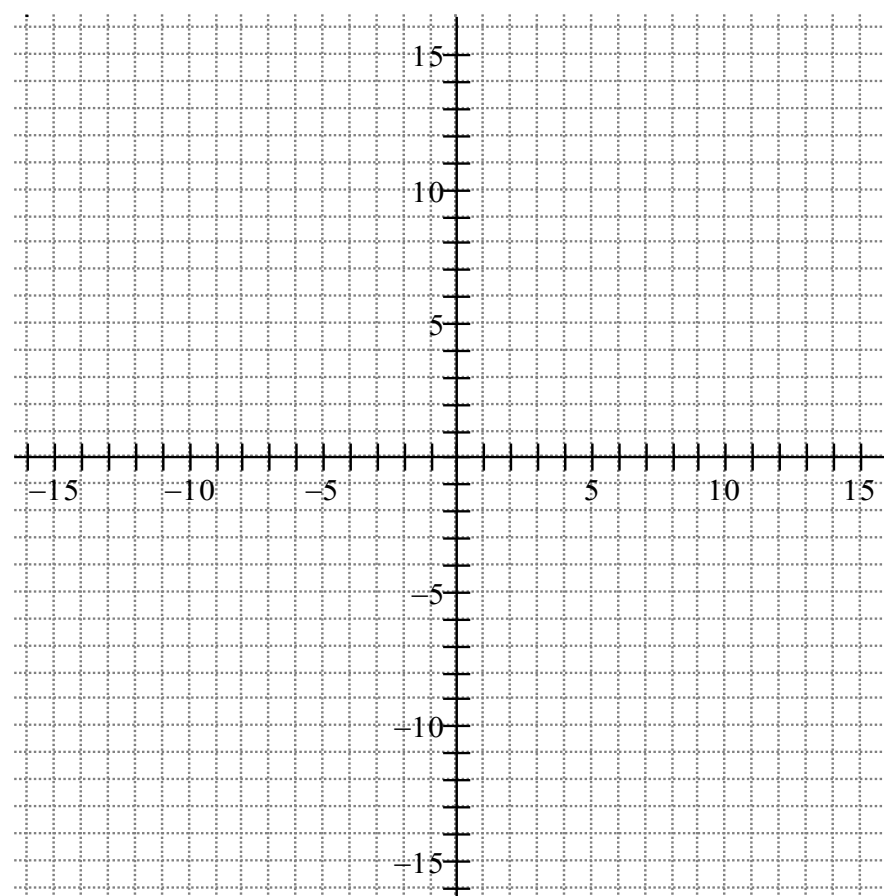
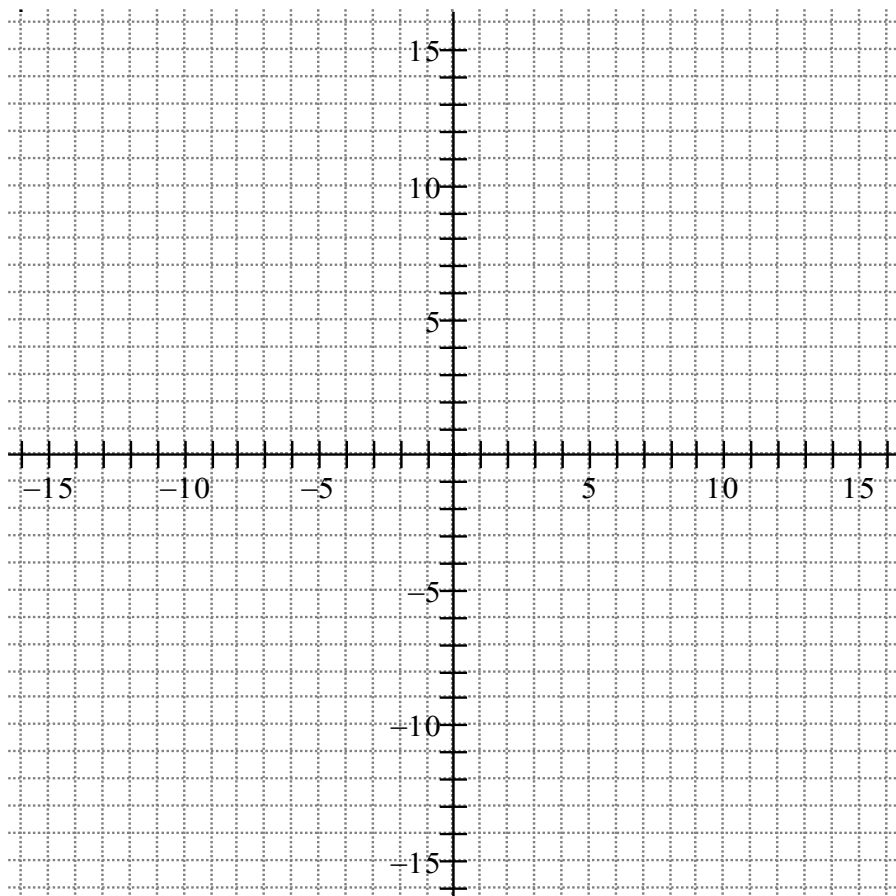
$$\vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

$$A\vec{t} = \begin{bmatrix} \\ \end{bmatrix}$$

$$A\vec{u} = \begin{bmatrix} \\ \end{bmatrix}$$

$$A\vec{v} = \begin{bmatrix} \\ \end{bmatrix}$$

$$A\vec{w} = \begin{bmatrix} \\ \end{bmatrix}$$



Matrix A does the following to every vector:

$$(2) \quad B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$$

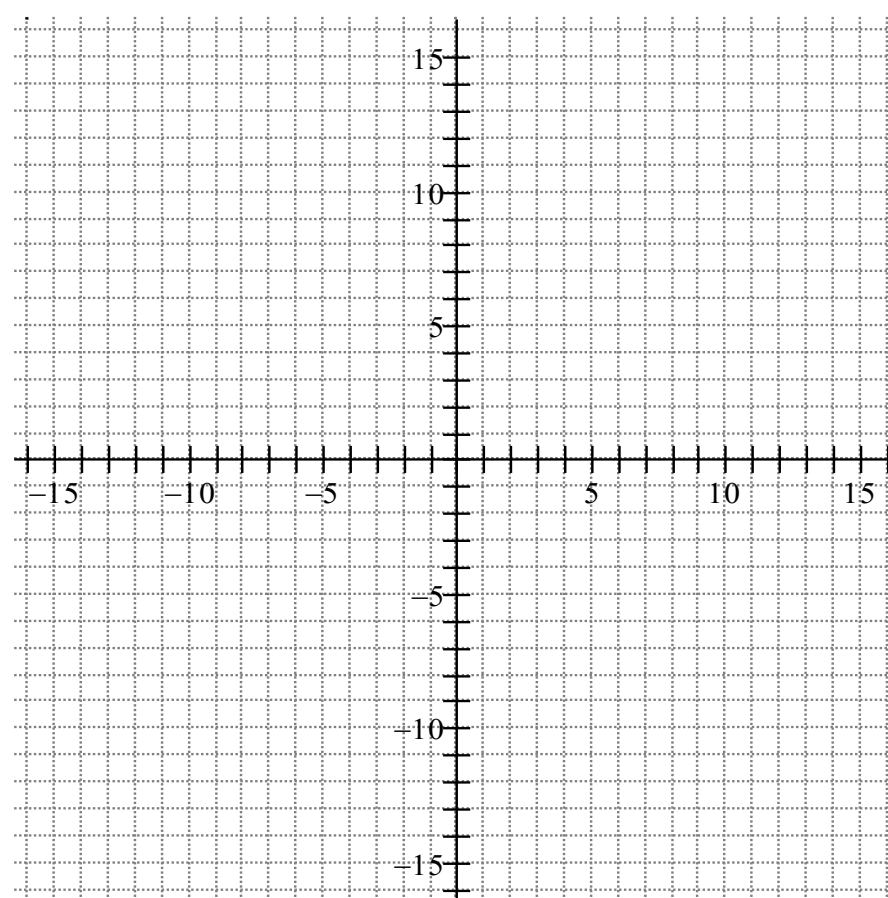
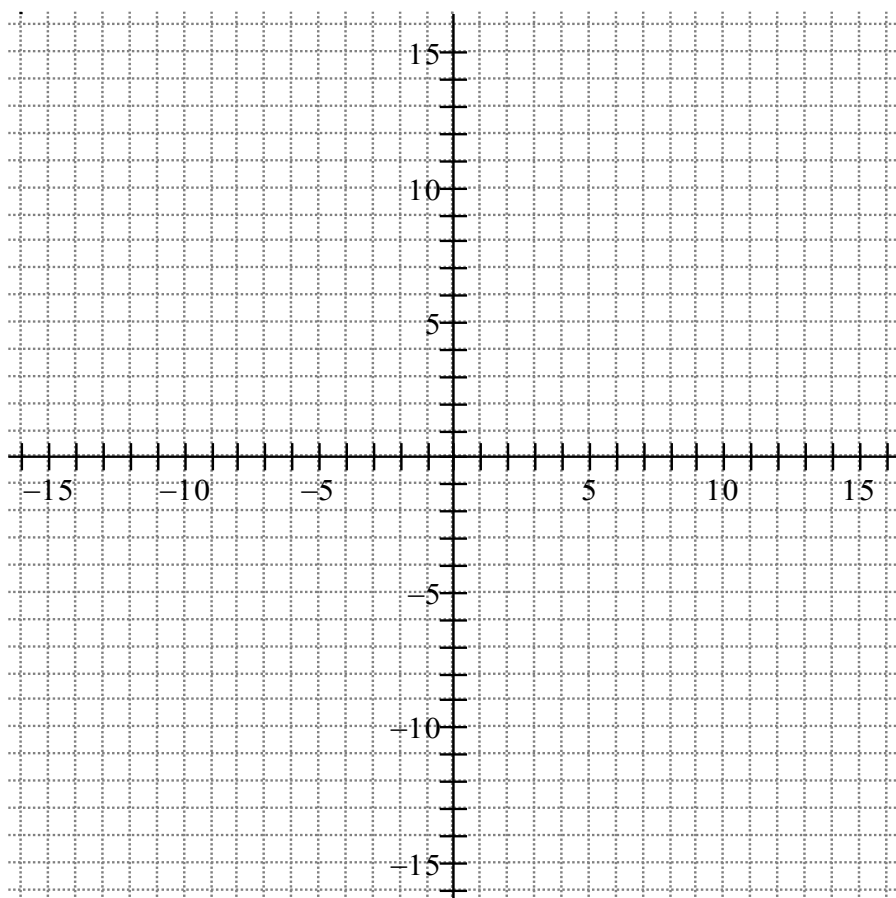
$$\vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

$$B\vec{t} = \begin{bmatrix} \\ \end{bmatrix}$$

$$B\vec{u} = \begin{bmatrix} \\ \end{bmatrix}$$

$$B\vec{v} = \begin{bmatrix} \\ \end{bmatrix}$$

$$B\vec{w} = \begin{bmatrix} \\ \end{bmatrix}$$



Matrix B does the following to every vector:

(3) $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$

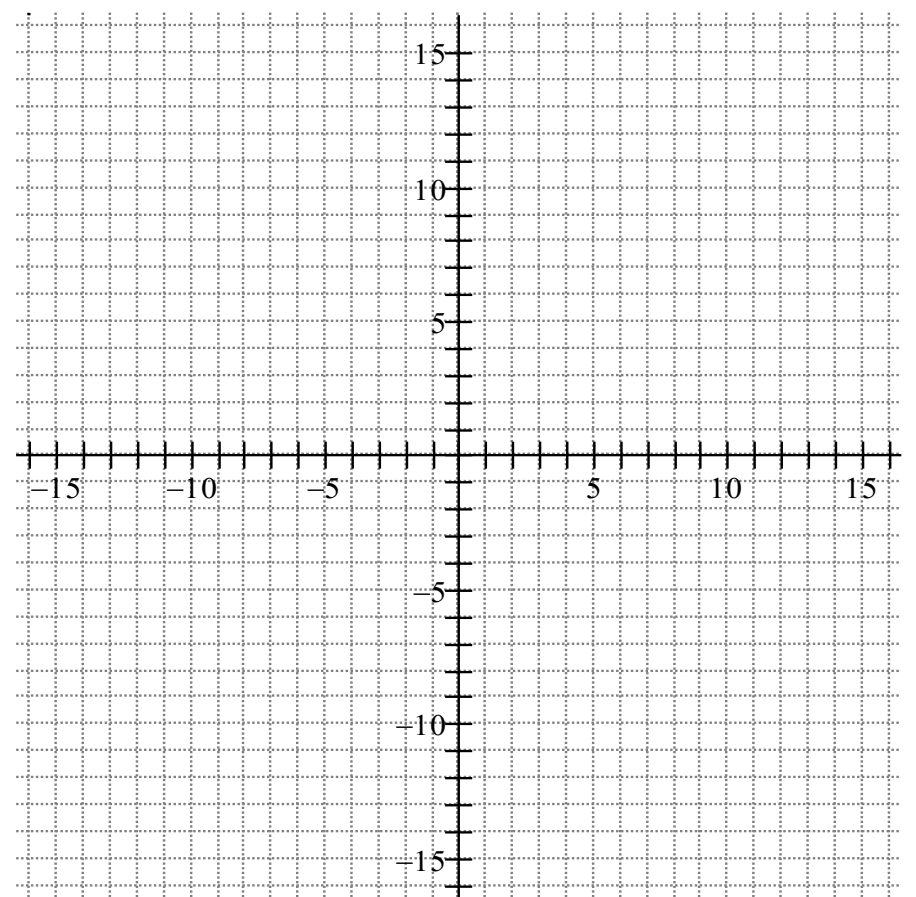
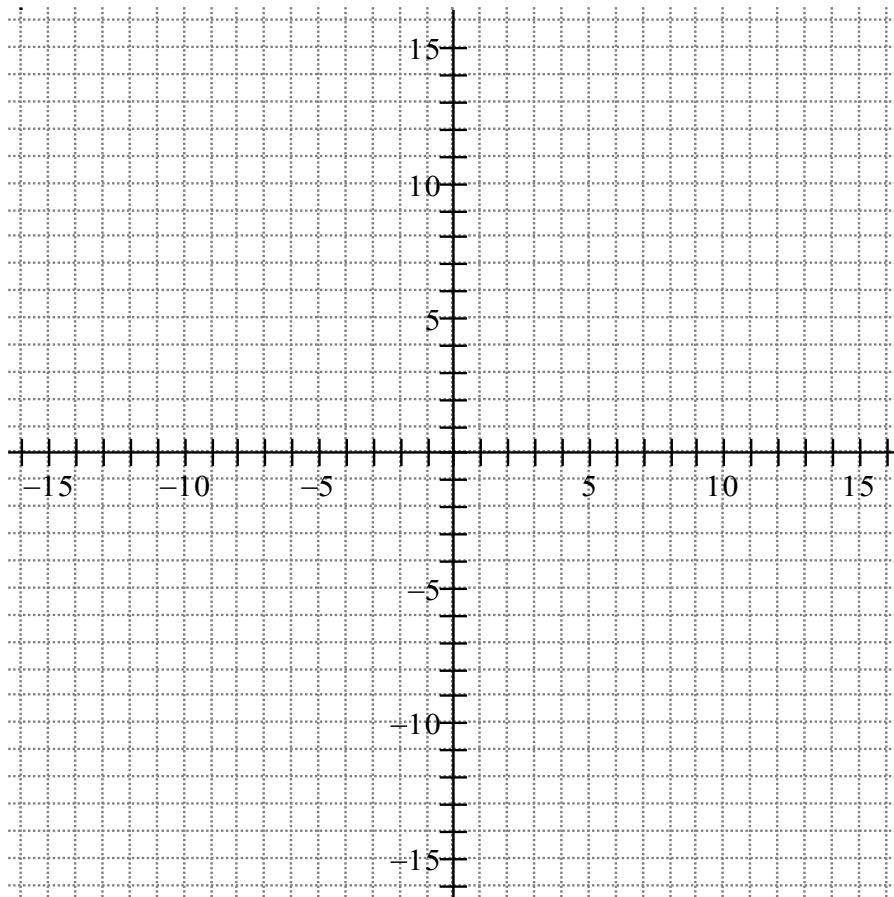
$\vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

$C\vec{t} = \begin{bmatrix} \\ \end{bmatrix}$

$C\vec{u} = \begin{bmatrix} \\ \end{bmatrix}$

$C\vec{v} = \begin{bmatrix} \\ \end{bmatrix}$

$C\vec{w} = \begin{bmatrix} \\ \end{bmatrix}$



Matrix C does the following to every vector:

(4) $D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$

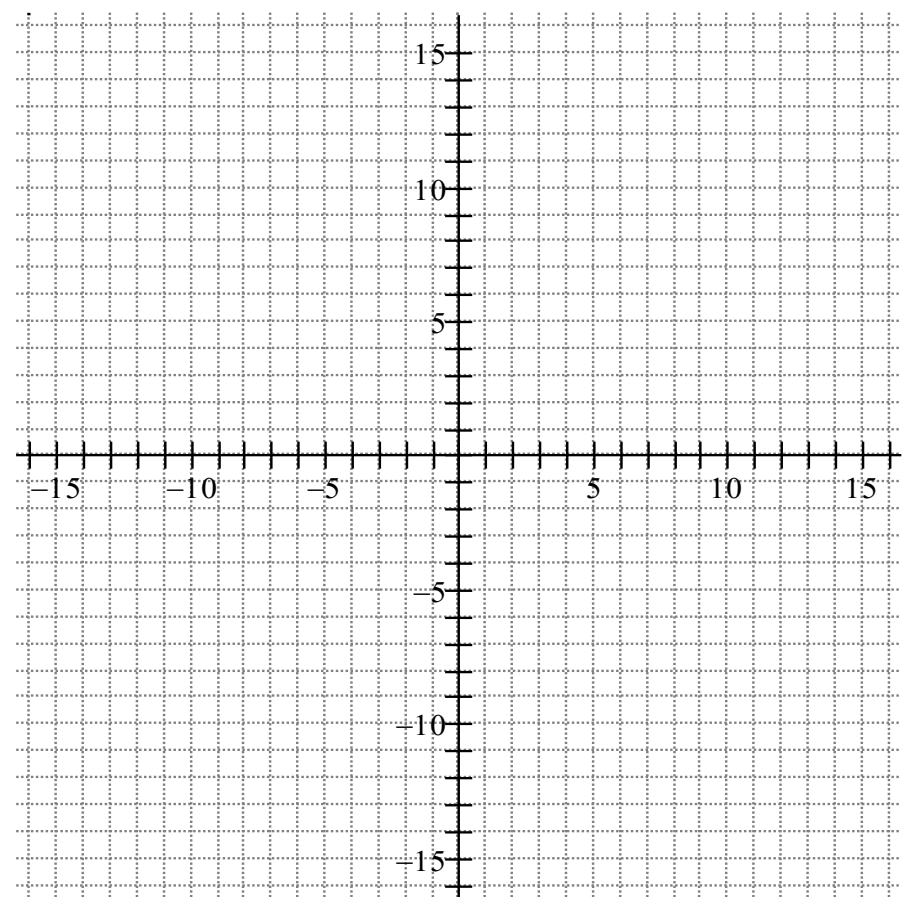
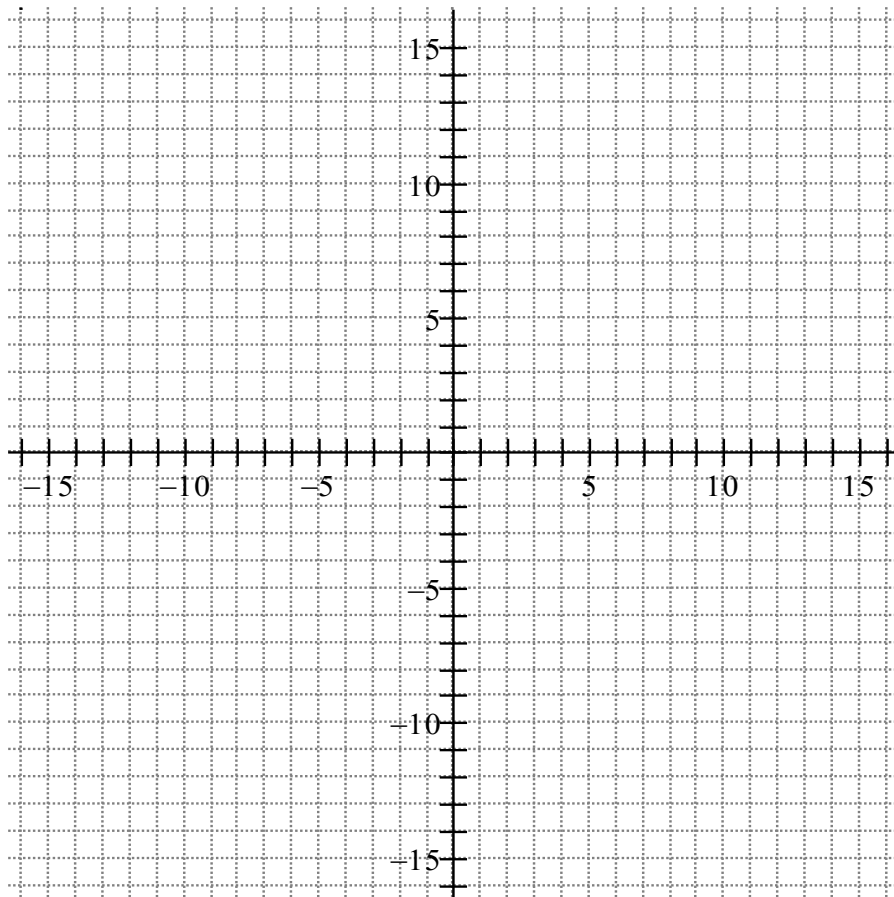
$\vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

$D\vec{t} = \begin{bmatrix} \\ \end{bmatrix}$

$D\vec{u} = \begin{bmatrix} \\ \end{bmatrix}$

$D\vec{v} = \begin{bmatrix} \\ \end{bmatrix}$

$D\vec{w} = \begin{bmatrix} \\ \end{bmatrix}$



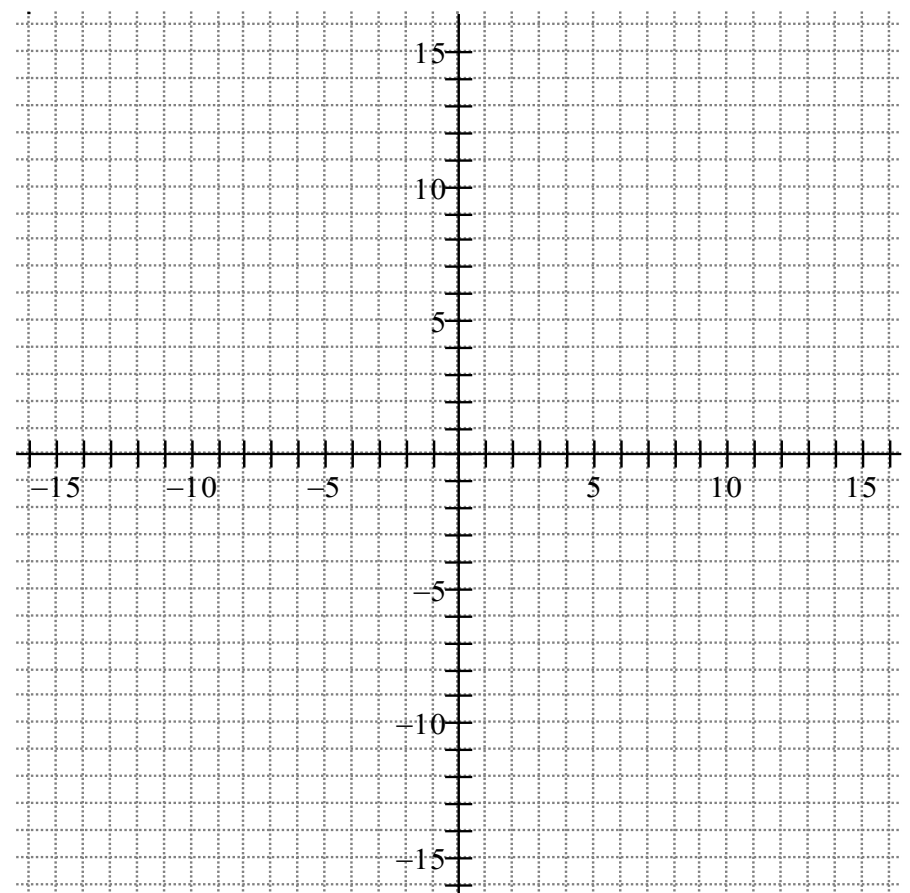
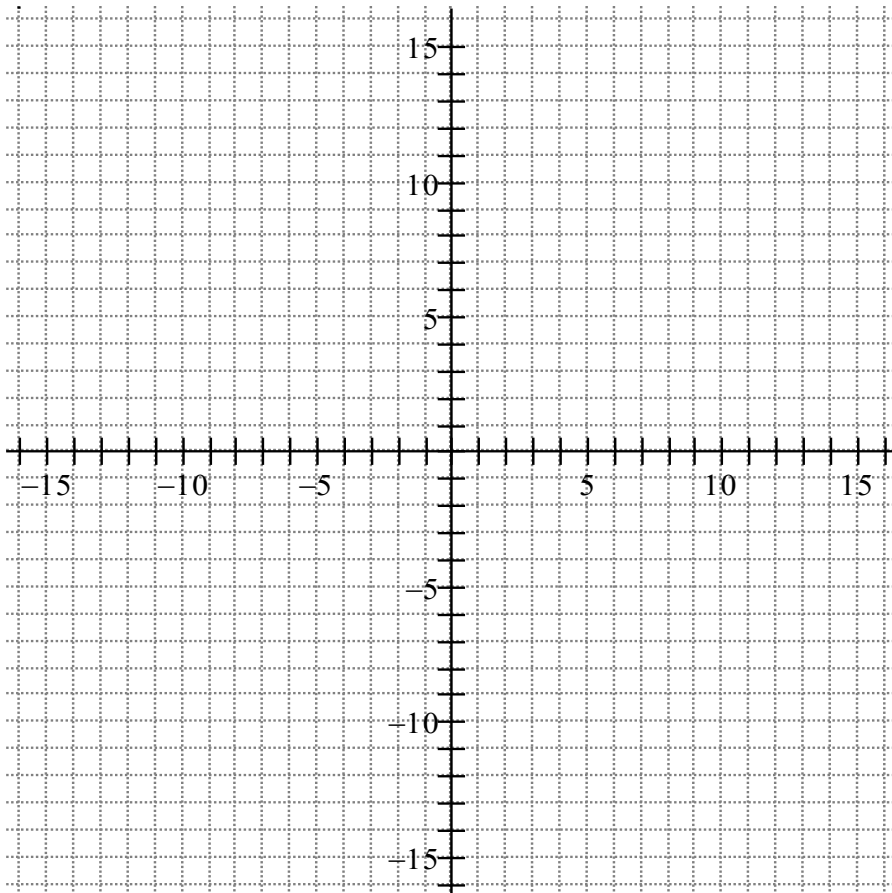
Matrix D does the following to every vector:

(5) $E = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$ graph this one as exactly as you can.

$\vec{t} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

$E\vec{t} = \begin{bmatrix} \\ \end{bmatrix}$ $E\vec{u} = \begin{bmatrix} \\ \end{bmatrix}$

$E\vec{v} = \begin{bmatrix} \\ \end{bmatrix}$ $E\vec{w} = \begin{bmatrix} \\ \end{bmatrix}$



Matrix E does the following to every vector

Part 3: Solve the following classic word problem, showing how to set it up, and explaining what you did to solve it. Use your own paper.

A TANGLED TALE

Knot IV

THE DEAD RECKONING

Lewis Carroll -- 1880

"I did dream of money-bags to-night."

Noonday on the open sea within a few degrees of the Equator is apt to be oppressively warm; and our two travelers were now airily clad in suits of dazzling white linen, having laid aside the chain-armor which they had found not only enduring in the cold mountain air they had lately been breathing, but a necessary precaution against the daggers of the banditti who infested the heights. Their holiday trip was over, and they were now on their way home, in the monthly packet which plied between the two great ports of the island they had been exploring.

Along with their armor, the tourists had laid aside the antiquated speech it had pleased them to affect while in knightly disguise, and had returned to the ordinary style of two country gentlemen of the Twentieth Century.

Stretched on a pile of cushions, under the shade of a huge umbrella, they were lazily watching some native fishermen, who had come on board at the last landing-place, each carrying over his shoulder a small but heavy sack. A large weighing-machine, that had been used for cargo at the last port, stood on the deck; and round this the fishermen had gathered, and, with much unintelligible jabber, seemed to be weighing their sacks.

"More like sparrows in a tree than human talk, isn't it?" the elder tourist remarked to his son, who smiled feebly, but would not exert himself so far as to speak. The old man tried another listener.

"What have they got in those sacks, Captain?" he inquired, as that great being passed them in his never ending parade to and fro on the deck.

The Captain paused in his march, and towered over the travelers -- tall, grave, and serenely self-satisfied.

"Fishermen," he explained, "are often passengers in My ship. These five are from Mhruxi -- the place we last touched at -- and that's the way they carry their money. The money of this island is heavy, gentlemen, but it costs little, as you may guess. We buy it from them by weight -- about five shilling a pound. I fancy a ten-pound note would buy all those sacks."

By this time the old man had closed his eyes -- in order, no doubt, to concentrate his thought on these interesting facts; but the Captain failed to realize his motive, and with grunt resumed his monotonous march.

Meanwhile the fishermen were getting so noisy over the weighing-machine that one of the sailors took the precaution of carrying off all the weights, leaving them to amuse themselves with such substitutes in the form of winch handles, belaying-pins &c., as they could find. This brought their excitement to a speedy end: they carefully hid their sacks in the folds of the jib that lay on the deck near the tourists, and strolled away.

When next the Captain's heavy footfall passed, the younger man roused himself to speak.

"*What* did you call the place those fellows came from, Captain?" he asked.

"Mhruxi, sir."

"And the one we are bound for?"

The Captain took a long breath, plunged into the word, and came out of it nobly. "They call it Kgovjni, sir."

"K -- I give it up!" the young man faintly said.

He stretched out his hand for a glass of iced water which the compassionate steward had brought him a minute ago, and had set down, unluckily, just outside the shadow of the umbrella. It was scalding hot, and he decided not to drink it. The effort of making this resolution, coming close on the fatiguing conversation he had just gone through, was too much for him: he sank back among the cushions in silence.

His father courteously tried to make amend for his *nonchalance*.

"Whereabouts are we now, Captain?" said he, "Have you any idea?"

The Captain cast a pitying look on the ignorant landsman. "I could tell you that, sir." he said, in a tone of lofty condescension, "to an inch!"

"You don't say so!" the old man remarked, in a tone of languid surprise.

"And mean so," persisted the Captain. "Why, what do you suppose would become of My ship, if I were to lose My Longitude and My Latitude? Could *you* make anything of My Dead Reckoning?"

"Nobody could, I'm sure!" the other heartily rejoined.

But he had overdone it.

"It's *perfectly* intelligible," the Captain said, in an offended tone, "to anyone that understands such things." With these words he moved away, and began giving orders to the men, who were preparing to hoist the jib.

Our tourists watched the operation with such interest that neither of them remembered the five money-bags, which in another moment, as the wind filled out the jib, were whirled overboard and fell heavily into the sea.

But the poor fishermen had not so easily forgotten their property. In a moment they had rushed to the spot, and stood uttering cries of fury, and pointing, now to the sea, and now to the sailors who had caused the disaster.

The old man explained it to the Captain.

"Let us make it up among us," he added in conclusion. "Ten pounds will do it, I think you said?"

But the Captain put aside the suggestion with a wave of the hand.

"No, sir!" he said, in his grandest manner. "You will excuse Me, I am sure; but these are my passengers. The accident has happened on board My ship, and under My orders. It is for Me to make compensation." He turned to the angry fishermen. "Come here, my men!" he said, in the Mhruxian dialect. "Tell me the weight of each sack. I saw you weighing them just now."

Then ensued a perfect Babel of noise, as the five natives explained, all screaming together, how the sailors had carried off the weights, and they had done what they could with whatever came handy.

Two iron belaying-pins, three blocks, six holy-stones, four winch handles, and a large hammer, were now carefully weighed, the Captain superintending and noting the results. But the matter did not seem to be settled, even then; an angry discussion followed, in which the sailors and the five natives all joined: and at last the Captain approached our tourists with a disconcerted look, which he tried to conceal under a laugh.

"It's an absurd difficulty," he said. "Perhaps one of you gentlemen can suggest something. It seems they weighed the sacks two at a time!"

"If they didn't have five separate weighings, of course you can't value them separately," the youth hastily decided.

"Let's hear all about it," was the old man's more cautious remark.

"They *did* have five separate weighings," the Captain said, "but -- Well, it beats *me* entirely!" he added, in a sudden burst of candor. "Here's the result. First and second sack weighed twelve pounds; second and third, thirteen and a half; third and fourth, eleven and a half; fourth and fifth, eight: and then they say they had only the large hammer left, and it took *three* sacks to weigh it down -- that's the first, third, and fifth -- and *they* weighed sixteen pounds. There, gentlemen! Did you ever hear anything like *that*?"

The old man muttered under his breath "If only my sister were here!" and looked helplessly at his son. His son looked at the five natives. The five natives looked at the Captain. The Captain looked at nobody: his eyes were cast down, and he seemed to be saying softly to himself, "Contemplate one another, gentlemen, if such be your good pleasure. I contemplate *Myself*!"