Math 253 Lab #2 **Parametric Equations and Polar Functions** **Names:** ______________________________

Work in groups of 2-4 for full credit and 5% bonus ______________________________
if turned in by Fri. of week 3. Make sure everyone ______________________________
in your group understands the question and its answer. ______________________________

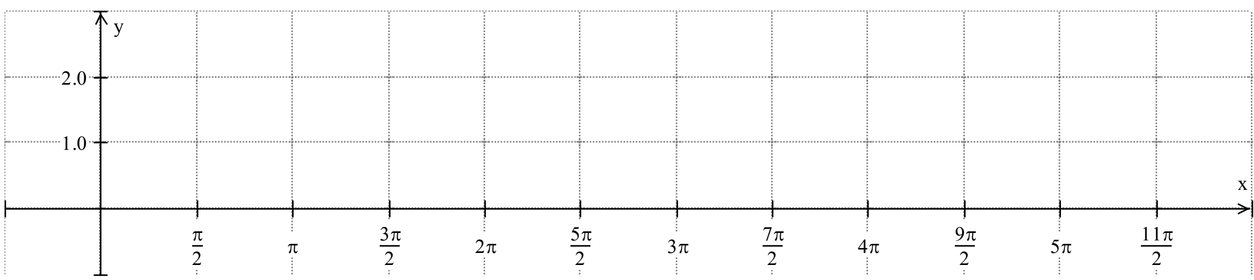
Work neatly and in pencil, with any additional work attached to this lab. Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. I will accept one lab this quarter as an individual project without penalty. Labs turned in after Friday are reduced by 25% and only accepted through week 4.

Part 1: Calculus of Parametric Functions

The Cycloid The general form for parametric equations of a cycloid is given by

$x = a(\theta - sin\theta)$ $y = a(1 - cos\theta)$ $a > 0$ We'll go ahead and let $a = 1$

1) Sketch a careful graph in the given window size for $\theta \in [0,6\pi]$ (In your calculator $\theta \leftrightarrow T$)



2) Find $\dfrac{dy}{dx} =$

and $\dfrac{d^2y}{dx^2} =$

3) Find the equations(s) of the tangent line at the point where $\theta = \frac{\pi}{6}$.

4) Find all points (if any) of horizontal tangency. What tells us where any horizontal tangents occur?

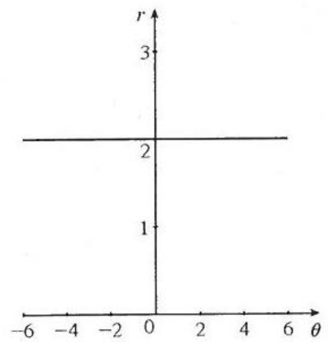
5) Find the area under one arc. Set up the integral and you may use your calculator to evaluate the integral. 3 bonus points if you show work and find the exact value!

6) Find the length of one arc of the curve. Set up the integral and you may use your calculator to evaluate the integral.

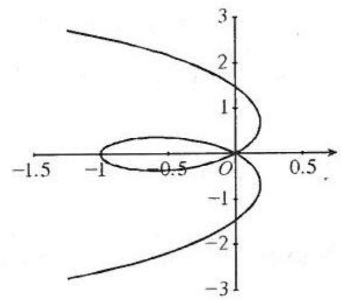
Part 2: Polar Functions

1. The graphs in the first column show how r varies with θ in rectangular form. The second column shows the resultant polar graphs of the functions in the first column. Match up the graphs so that each pair corresponds to one equation for r as a function of θ. Write the equation for each graph in the second column in the form $r = f(\theta)$, in the space provided.

1.

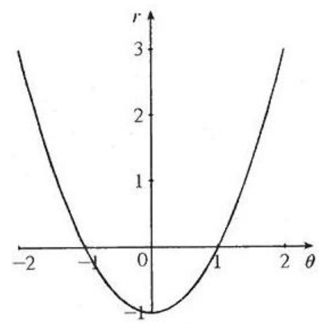


A.

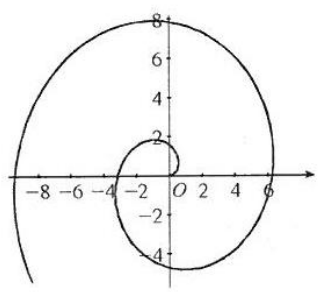


$r =$

2.

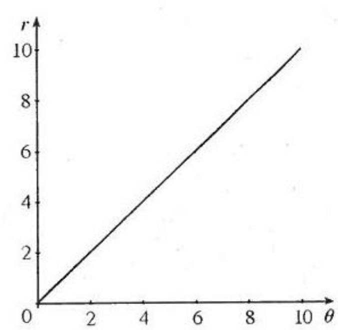


B.

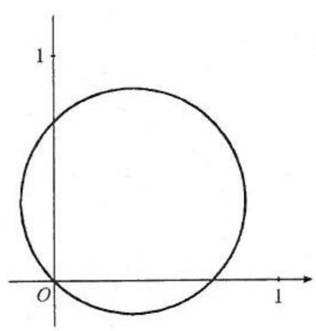


$r =$

3.

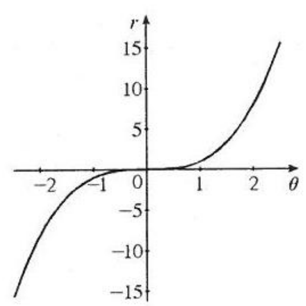


C.

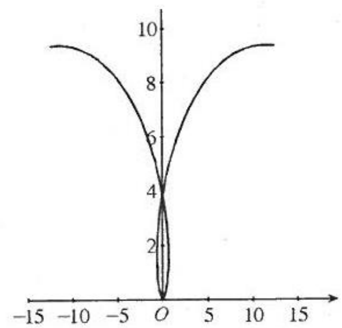


$r =$

4.

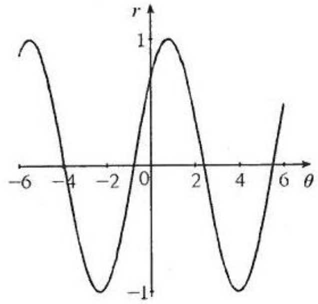


D.

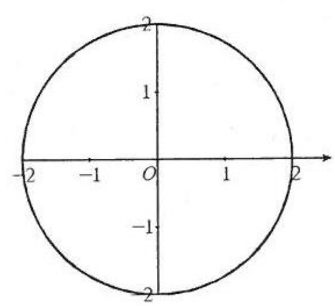


$r =$

5.



E.

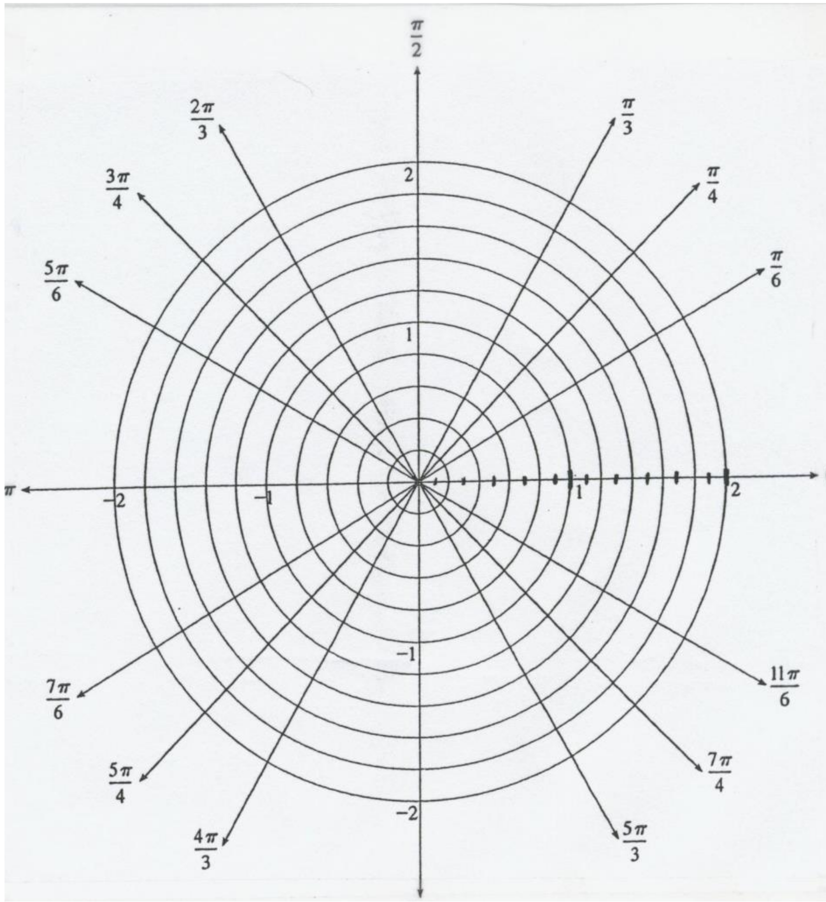


$r =$

2. Cardioids – **a)** Consider the polar curve $r = 1 + \cos(\theta)$

Carefully sketch the graph of the curve by completing the table, plotting, and connecting the points.

r	θ
	0
	$\frac{\pi}{6}$
	$\frac{\pi}{4}$
	$\frac{\pi}{3}$
	$\frac{\pi}{2}$
	$\frac{2\pi}{3}$
	$\frac{3\pi}{4}$
	$\frac{5\pi}{6}$
	π
	$\frac{7\pi}{6}$
	$\frac{5\pi}{4}$
	$\frac{4\pi}{3}$
	$\frac{3\pi}{2}$
	$\frac{5\pi}{3}$
	$\frac{7\pi}{4}$
	$\frac{11\pi}{6}$
	2π



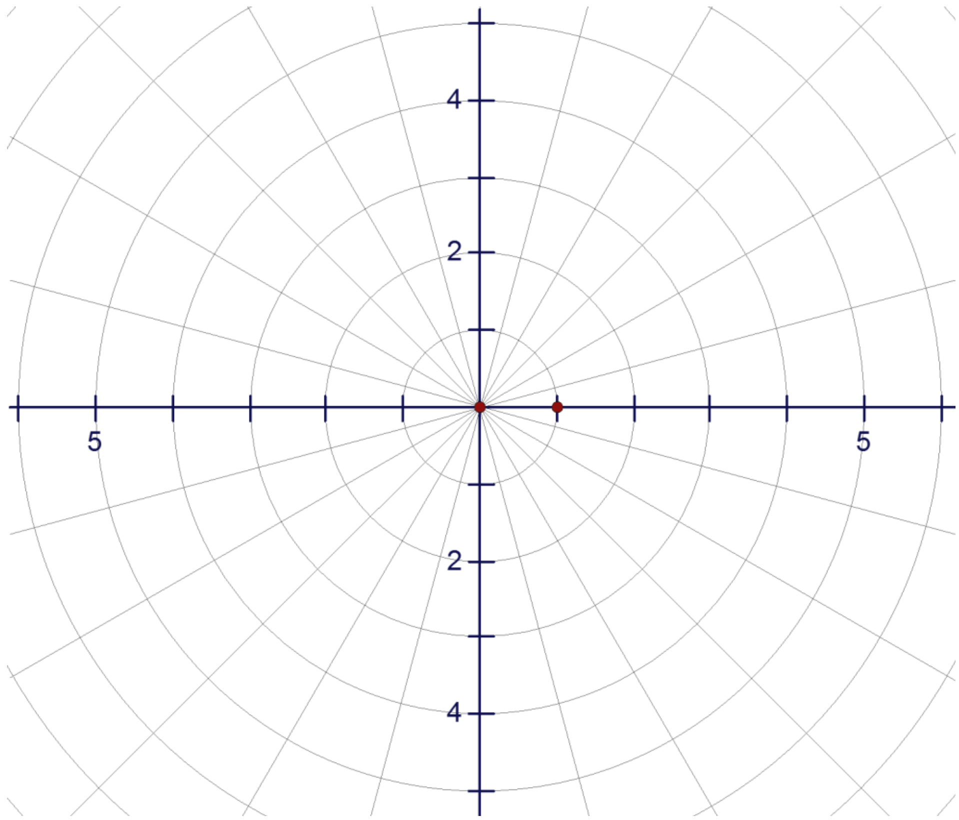
b) Find the polar coordinates for the points where the tangent line is ***horizontal***. Find exact values (no decimals, please!). Show work on attached sheet, if needed.

c) Find the polar coordinates for the points where the tangent line is ***vertical***. Find exact values (no decimals, please!). Show work on attached sheet, if necessary.

d) Carefully draw the vertical and horizontal tangents on the graph.

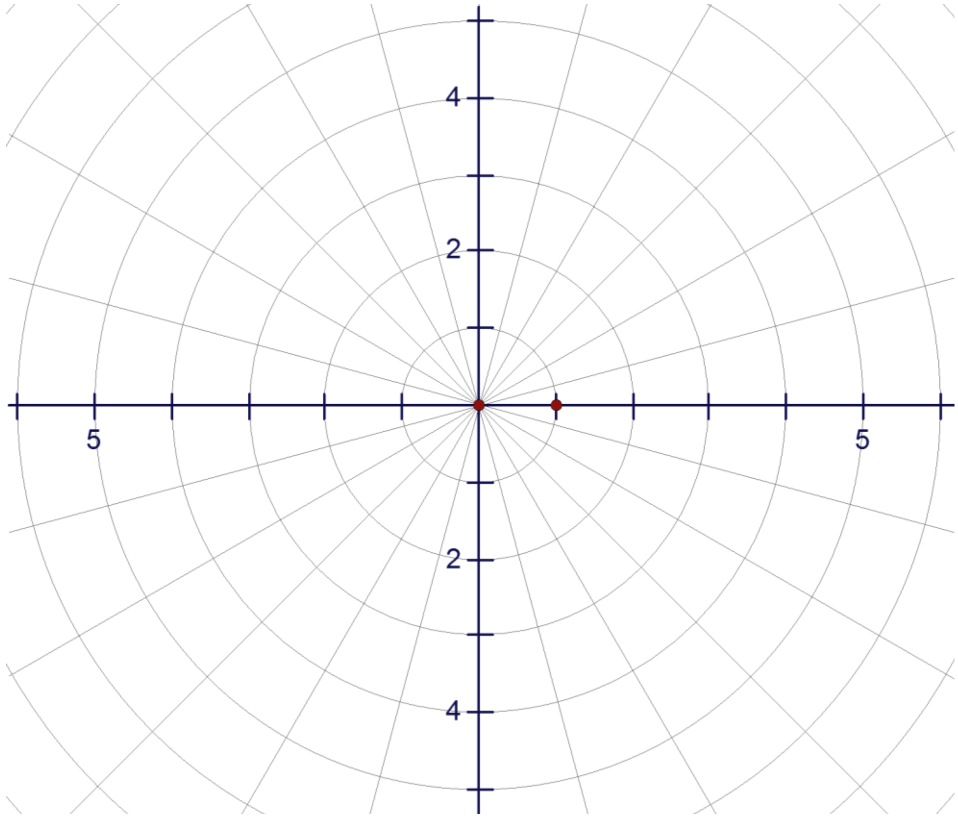
Part 3 Area of Polar Regions

1. Carefully sketch and label the graphs of the curves $r = -6\cos\theta$ and $r = 2 - 2\cos\theta$, then very lightly shade in the area of the region created by the intersection of the two regions bounded by the curves. Find the area of the shaded region. Think about finding the area in pieces and use symmetry to help. (Note, on the polar graph provided the increments for θ are $\frac{\pi}{12}$)



2. The radiation from a transmitting antenna is not uniform in all directions. The intensity from a particular antenna is modeled by $r = a\cos^2\theta$.

a) Graph and label the model for a = 4 and a = 6.



b) Find the area of the geographic region between the two curves in part a).