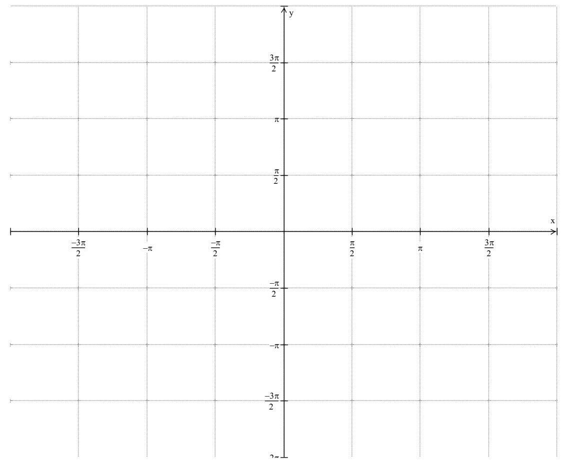
Math 253 Lab #1 Parametric Equations Names: ______________________________

Work in groups of 2-4 for full credit and 5% bonus ______________________________
if turned in by Fri. of week 2. Make sure everyone ______________________________
in your group understands the question and its answer. ______________________________
Work neatly and in pencil, with any additional work attached to this lab. Your score will depend upon: Neatness, Clarity, Organization, Thoroughness, and Correctness. I will accept one lab this quarter as an individual project without penalty.

Part 1: A Cool Trick with Parametric Graphing

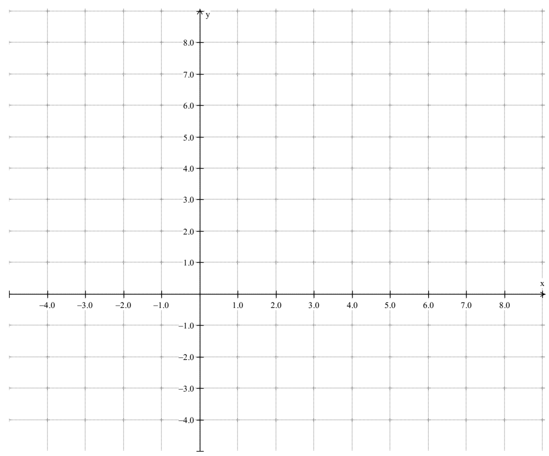
1) Consider these two sets of parametric equations:

$x = t$		$x = \sin(t)$
$y = \sin(t)$	**and**	$y = t$
$-2\pi \le t \le 2\pi$		$-2\pi \le t \le 2\pi$



Sketch both graphs to the right, and then explain (using a complete sentence) their relationship to each other.

2) Using a graphing calculator to help, make a good sketch of $f(x) = x^5 - 3x^3 + 5x + 2$. How do you know it is one-to-one? If a function is one-to-one, what does that tell us?



Hint:
Tmin = -3
Tmax = 3
Tstep = .02
xmin = -5
xmax = 9.1
xscl = 1
ymin = -5
ymax = 9.1

3) Try to find $f^{-1}(x)$ algebraically by hand. Explain (in words) where you get stuck.

4) Explain how to use a graphing calculator to graph $f^{-1}(x)$. Sketch and label $f^{-1}(x)$ on the same graph as *f*.

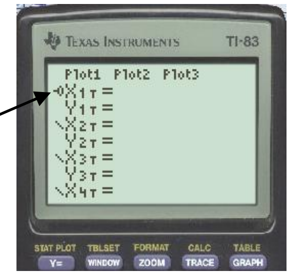
Part 2: Round and Round

Consider the circle $x^2 + y^2 = 4$. We want to construct parametric curves that will trace this circle in different ways. Start with a parametric function of the form:

$$x = a_1 \cos(a_2 t)$$
$$y = a_3 \sin(a_4 t)$$
$$0 \le t \le 2\pi \ (or\ \infty) \qquad \text{where } a_n \in \mathbb{R}$$

This means that the a_n's can be any real number.

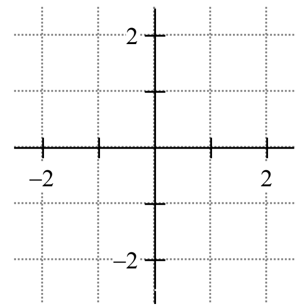
Use line type-“follow the ball”



(1) With $a_2 = a_4 = 1$ find a_1 and a_3 so that every point (x,y) lies on the circle $x^2 + y^2 = 4$. We'll use $0 \le t \le 2\pi$

a_1 = a_3 =

(2) Describe the motion of the particle if $a_2 = a_4 = 1$.
Use a <u>sketch</u>, and label the start and end points.
Use a <u>sentence</u> to describe the motion.



(3) Describe the motion of the particle if $a_2 = a_4 = -1$. Use a <u>sentence</u> to describe the motion.

(4) Describe the motion of the particle if $a_2 = a_4 = 2$. Use a <u>sentence</u> to describe the motion.

(5) Find a_2 and a_4 so that the particle goes counterclockwise five times around the circle, starting at (2,0).

a_2 = and a_4 =

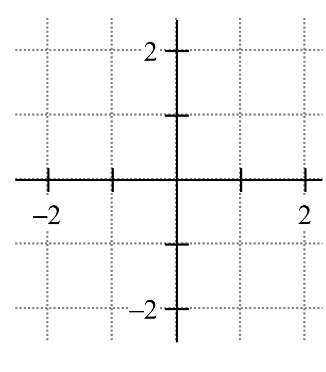
(6) Find a_2 and a_4 so that the particle goes clockwise three times around the circle, starting at (2,0).

a_2 = and a_4 =

Note: these parametric functions give us curves called Lissajous figures, and are used in electrical engineering to see if two signals are "in sync". They can also be used in music to show whether a musical interval is in tune.

(7) Graph x = 2cos (1t), y = 2sin (2t) on your calculator, graphing it to the right, and labeling the start and end points. Use a <u>sentence</u> to describe the motion.

<u>Hint:</u>
Tmin = 0
Tmax = 2pi
Tstep = pi/24
xmin = -4.7
xmax = 4.7
xscl = 1
ymin = -3.7
ymax = 3.7



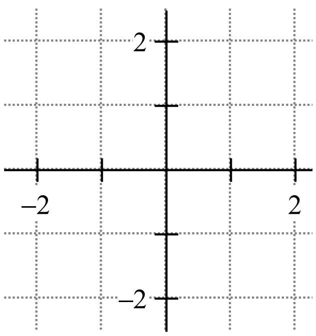
(8) Graph x = cos (2t), y = sin (4t) on your calculator. How is it the same as the previous graph, and how is it different? Use a <u>sentence</u> to describe the motion.

(9) Graph x = cos (3t), y = sin (6t) on your calculator. How is it the same as the previous two graphs, and how is it different? Use a <u>sentence</u> to describe the motion.

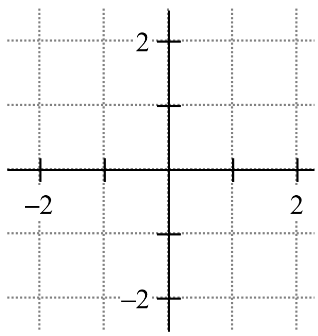
For x = 2cos ($a_2 t$), y = 2sin ($a_4 t$)

Fix $a_2 = 1$, and graph the funcitons with $a_4 = 1, 2, 3$.

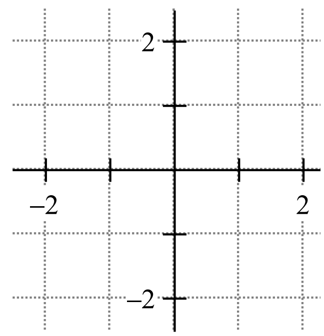
$a_4 = 1$



$a_4 = 2$

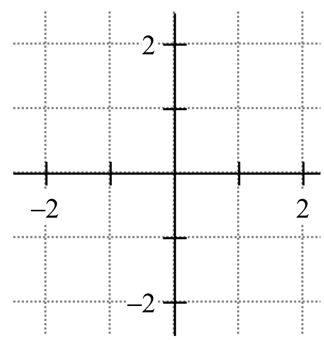


$a_4 = 3$

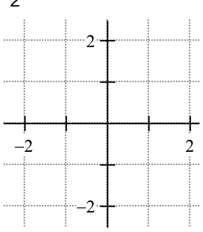
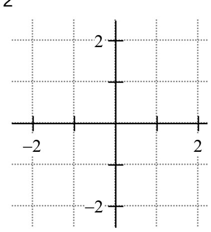
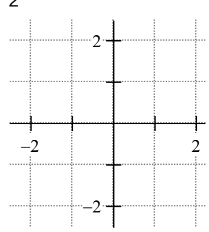
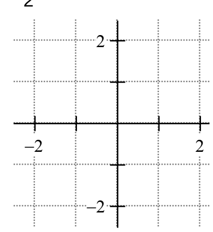


Describe what happens as a_4 increases. Use a <u>sentence</u>.

Predict what the figure will look like if $a_2 = 1$ and $a_4 = 5$.
Include a careful sketch.



(10) Now fix $a_4 = 1$, and look (on your calculator) at graphs for $a_2 = 1, 2, 3, 4$. What happens as a_2 increases? Describe the effect carefully, using graphs to illustrate.

<table>
<tr><th colspan="2">a_2 is odd</th><th colspan="2">a_2 is even</th></tr>
<tr><td>$a_2 = 1$
</td><td>$a_2 = 3$
</td><td>$a_2 = 2$
</td><td>$a_2 = 4$
</td></tr>
<tr><td colspan="2">If a_2 is odd, you get ...</td><td colspan="2">If a_2 is even, you get ...</td></tr>
</table>

Part 3: Can Curves Fill Space?

Consider the parametric curve given by:

$$x(t) = 2\cos(\sqrt{2}\,t)$$
$$y(t) = 1.5\sin(\sqrt{3}\,t)$$

1) Using the window $-2 \le x \le 2$ and $-1.5 \le y \le 1.5$, graph this parametric curve for $0 \le t \le 13$. What set of points will be "hit" by this curve?

2) Next graph this curve for $0 \le t \le 100$. What is your guess now?

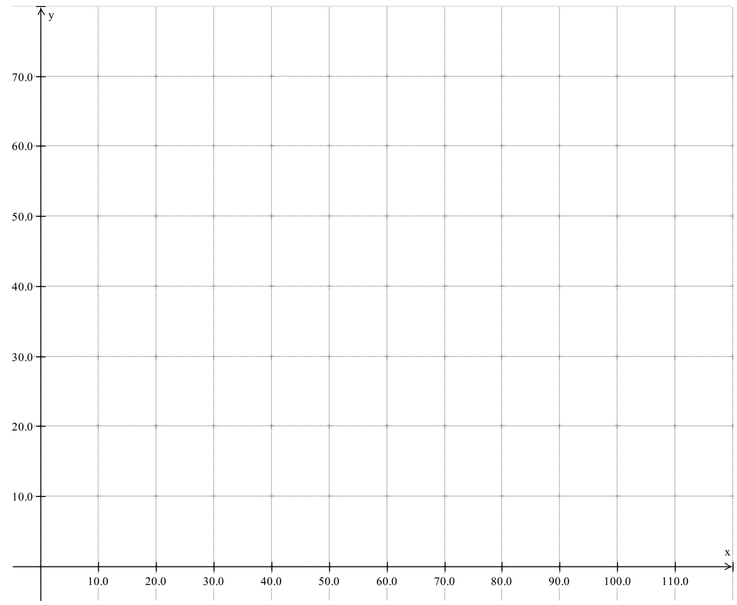
3) Finally, graph this curve for $0 \le t \le 200$. What is your guess this time? How is your guess related to the rectangle $-2 \le x \le 2$ and $-1.5 \le y \le 1.5$?

4) This parametric curve is called a "space filling" curve since if we extend the domain of t far enough, it can be made to come as close as we like to any desired point in the rectangle. Try this exercise again, replacing $\sqrt{2}$ with $\sqrt{5}$ and $\sqrt{3}$ with $\sqrt{7}$. Now try it again with $\sqrt{2}$ and $\sqrt{18}$. Given any $\sqrt{p}$ and $\sqrt{q}$, how can you tell whether or not the curve is space filling?

5) Think about the examples on this page and the previous pages – which pictures do you think show that electrical signals are in sync, and which pictures show that electrical signals are not in sync?

Part 4 An application:

A jungle wildlife preserve extends 80 miles north and 120 miles east of a ranger station. The ranger leaves from a point 100 mile east of the station along the southern boundary to survey the area. He travels 0.6 miles north and 0.5 miles west every minute. A lion leaves the west edge of the preserve 51 miles north of the station at the same time the ranger leaves his original position. Every minute the lion moves 0.1 miles north and 0.3 miles east.



1. Sketch the paths of the lion and the ranger on the same piece of graph paper. Clearly indicate their starting positions.
2. Write linear equations of the form $y = mx + b$ for the paths traveled by the ranger and the lion.

3. At what point do their paths cross? Indicate on the sketch of the paths.

4. How far from each starting point is the point where their paths cross? (Think Pyth. Theorem)

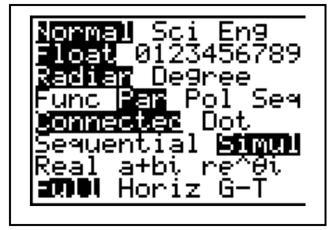
5. At what rates are the lion and the ranger each traveling per minute? (Think Pyth. Theorem)

6. How long does it take for the lion and the ranger to reach the point at which their paths cross?

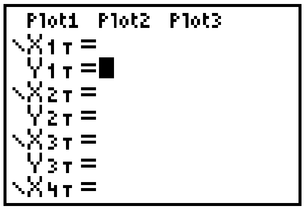
7. For each time given in the chart indicate the x and y coordinates of the ranger and the lion relative to the ranger station.

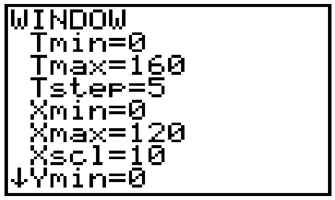
	Ranger			Lion	
Time (t-min)	x	y		x	y
0					
1					
2					
5					
10					
t					

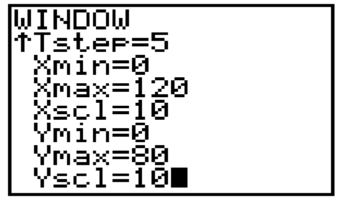
8. From the table, the parametric equations that model the motion of the ranger and the lion are found in the last row. To graph parametric equations, press[Mode] and select **Par** and **Simul**



Press [Y=] and enter the ranger's equations in X_{1T} and Y_{1T} and the lions equations in X_{2T} and Y_{2T}. Then Press [Window] and set the graphing values as shown.







9. Trace to the point at which the two paths cross. What are the coordinates of the point of intersection of the two paths?

10. At what time does the ranger reach that point?

11. At what time does the lion reach that point?

12. Do the times agree with the answers to question 6? Do the ranger and the lion collide? How do you know?

13. Hopefully you found that they don't collide. Now find the distance between the ranger and the lion at the following times. Copy the information from the table in question 7. Recall the distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

	Ranger		Lion		Distance
Time (t-min)	x	y	x	y	d
0					
1					
2					
5					
10					
t					

14. The distance found as a function of time can be graphed parametrically. Enter T as X_{3T} and the distance equation as Y_{3T}. Use this graph to determine how close the lion got to the ranger and when were they the closest. The graph is given in 5 minute increments. Change Tstep to 1 to get accuracy to the nearest minute.