

Show all necessary work for full or partial credit, neatly and in **pencil only**. Use correct notation. The test is worth 100 points but there are 110 points available.

1. 32pts Fill in the missing parts – blanks for parts a) thru d) – using exact values in the form of simplified radicals, fractions, or whole numbers not decimals.

- When necessary: write the angle measure, θ , in **both degree and radian** using only the **standard angles** in the interval $[0^\circ, 360^\circ)$ and $[0, 2\pi)$;
- Identify the reference angle, $\bar{\theta}$, in degrees, note $0^\circ < \bar{\theta} < 90^\circ$;
- **Draw the standard angle**, θ , in **standard position** using the “indicator” curve with an arrow, see b);
- Label the ordered pair representing the point of intersection with the angle and the given unit circles (like part b).

a) (8 pts) $\theta = \frac{7\pi}{4} = 315^\circ$

$\bar{\theta} = 45^\circ$

$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

$\sin \theta = -\frac{\sqrt{2}}{2}$ $\cos \theta = \frac{\sqrt{2}}{2}$ $\tan \theta = -1$

b) (7 pts) $\theta = 210^\circ = \frac{7\pi}{6}$

$\bar{\theta} = 30^\circ$

$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\sin \theta = -\frac{1}{2}$ $\cos \theta = -\frac{\sqrt{3}}{2}$ $\tan \theta = \frac{1}{\sqrt{3}}$

c) (9 pts) $\theta_c = -300^\circ$ is coterminal with which standard angle? Draw the standard angle.

$\theta = 60^\circ = \frac{\pi}{3}$

$\bar{\theta} = 60^\circ$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\sin \theta = \frac{\sqrt{3}}{2}$ $\cos \theta = \frac{1}{2}$ $\tan \theta = \sqrt{3}$

d) (8 pts) $\theta = 90^\circ = \frac{\pi}{2}$

$\bar{\theta} = \text{N.A.}$

$(0, 1)$

$\sin \theta = 1$ $\cos \theta = 0$ $\tan \theta = \text{undefined}$

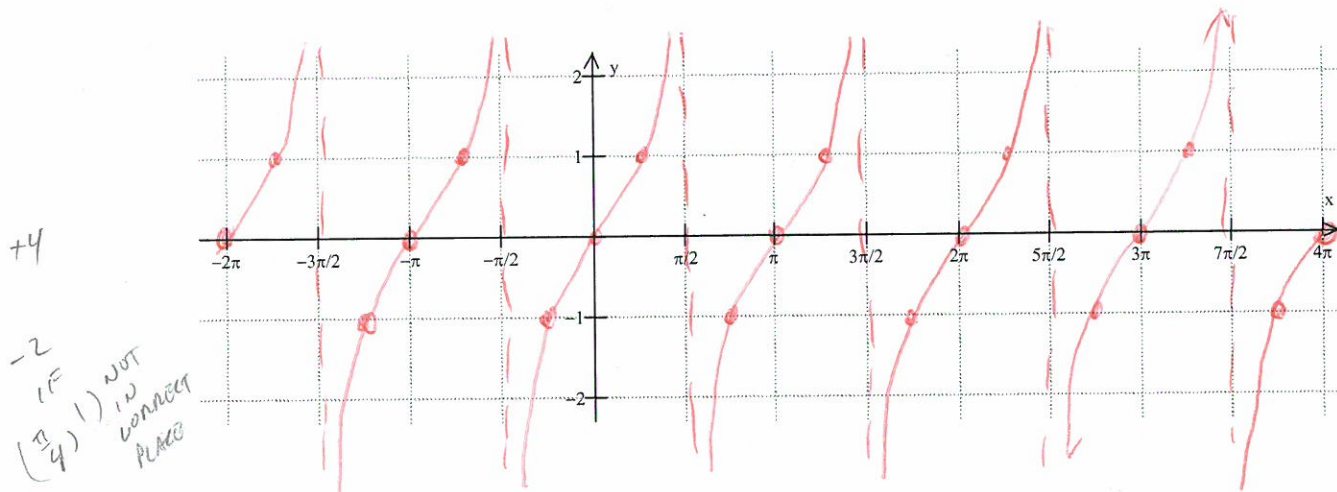
2. 16pts Which of the three main trig functions, sine, cosine, tangent, has a graph with asymptotes?

+2 tangent

Sketch the graph of the function you chose. Make sure that any zeros, maximums, minimums, or other key points, and asymptotes are correctly placed.

-8 FOR OTHER GRAPH

Sketch its graph all the way from the left to the right. Use radians.



What is the domain (what inputs are allowed)?

+4 $\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}\}$

What is the range (what outputs can be expected)?

+4 $y \in \mathbb{R}$

+2 What is the period of the function (how long before it repeats)?

π

3. 12 pts Given the shaded area of the sector enclosed by a central angle of 140° is 16 in.^2 , find the radius of the circle and the arc length S that subtends the angle. Use appropriate units and round to two decimals - hundredths.

HINT $S = r\theta$ & $A = (\frac{1}{2})\theta r^2$

$\theta = 140^\circ (\frac{\pi}{180^\circ})$

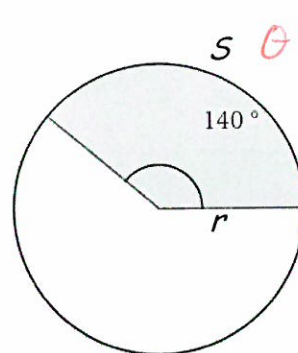
$$16 = \frac{7\pi}{18} r^2$$

$$\frac{16 \cdot 18}{7\pi} = r^2$$

$$r = \sqrt{\frac{16 \cdot 18}{7\pi}}$$

$$r = \sqrt{\frac{16 \cdot 18 \pi}{7}} \approx 11.37$$

$$S = r \left(\frac{7\pi}{9} \right)$$



$$\theta = \frac{7\pi}{9}$$

+6

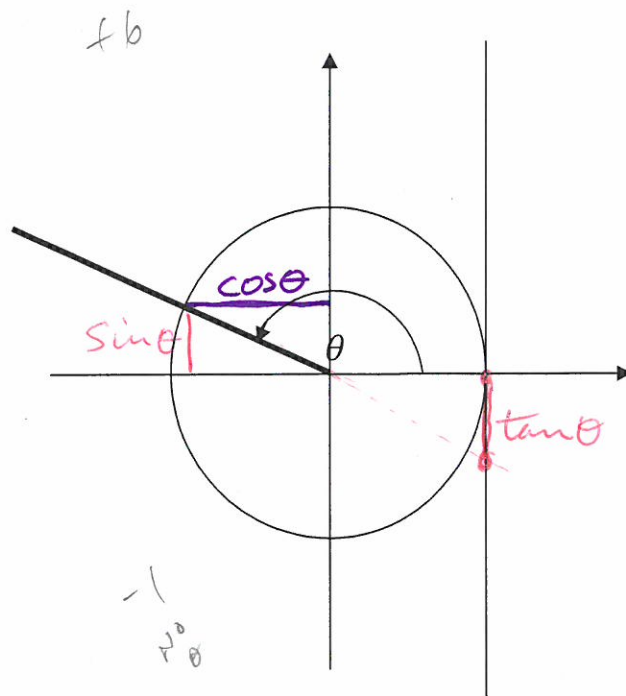
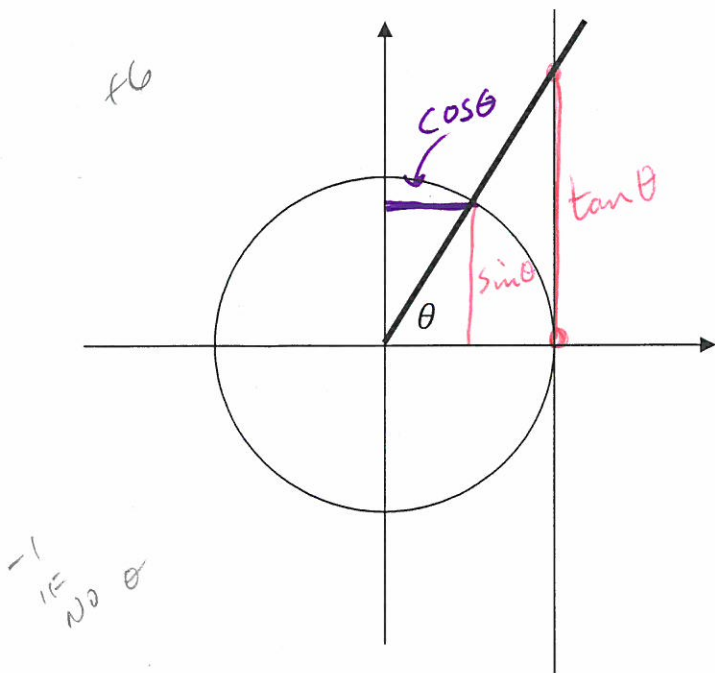
+6

OK IF 27.78 in

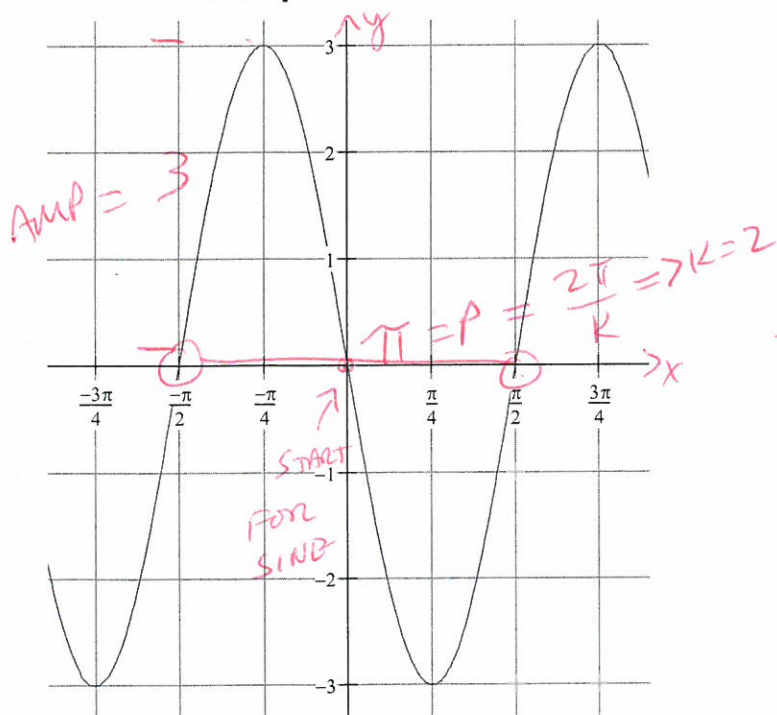
$r = 3.62 \text{ in.}$

$S = 8.84 \text{ in.}$

4. 12 pts Draw and label the line segments that represent $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ in the pictures of the unit circle below:



5. 12pts Give a possible equation for each graph, you may use your calculator as a check. The parameters must be in exact form, no decimals: $p = 2\pi \rightarrow k = 1$



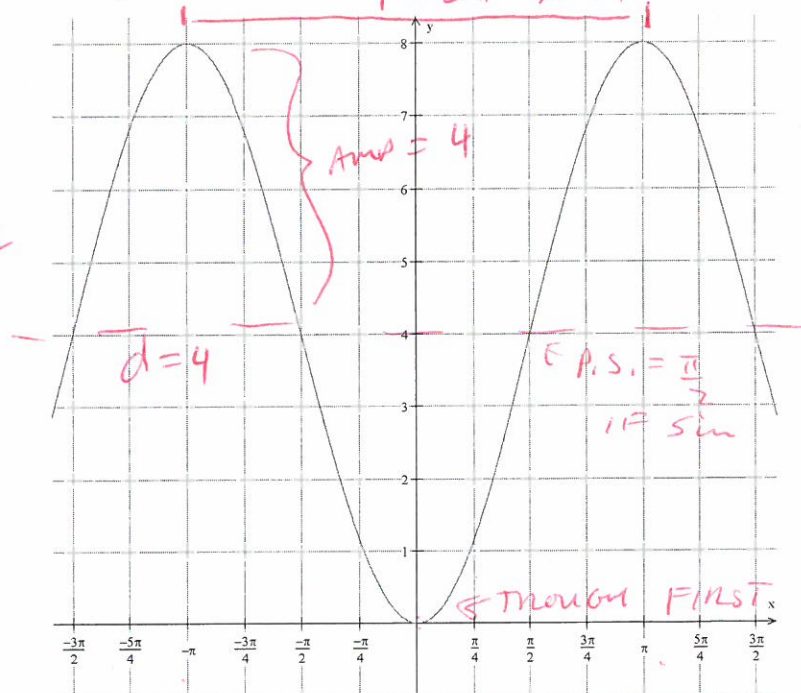
$y = -3 \sin(2x)$ f6

or

$y = 3 \sin(2(x + \frac{\pi}{2}))$

or $y = 3 \cos(2(x + \frac{\pi}{4}))$

-2
1 = NO
y =



f6 $y = -4 \cos(x) + 4$

or

$y = 4 \sin(x - \frac{\pi}{2}) + 4$

or

$y = 4 \cos(x + \pi) + 4$

6. 16pts For the given functions: Sketch each graph, showing at least two full cycles, clearly labeling the scale on the x and y axes and clearly plot the 5 key points. Identify the amplitude, period and phase shift. Use radians, not degrees.

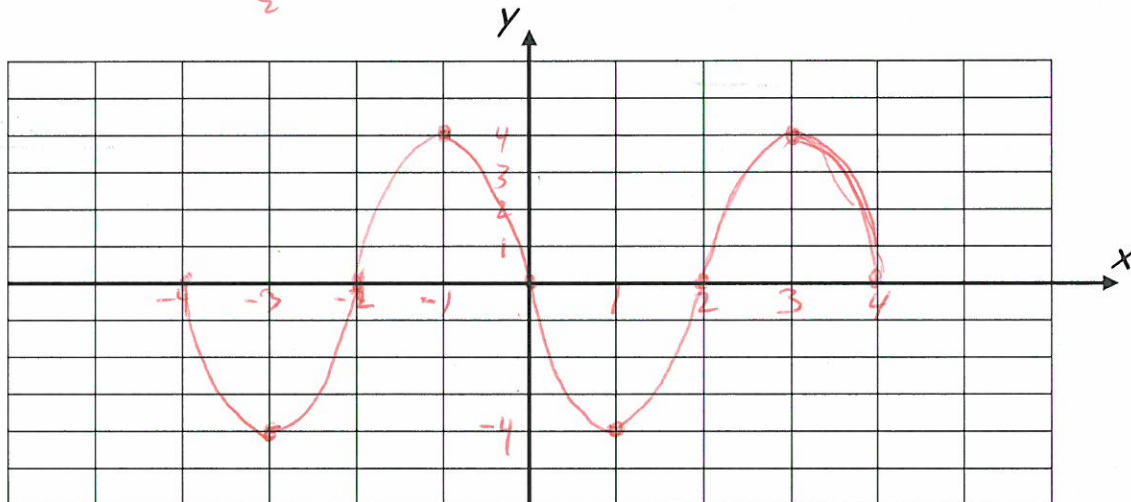
a) $y = -4\sin\left(\frac{\pi}{2}x\right)$

$P = \frac{2\pi}{\frac{\pi}{2}} = 4$

amplitude= 4

period= 4

phase shift= 0

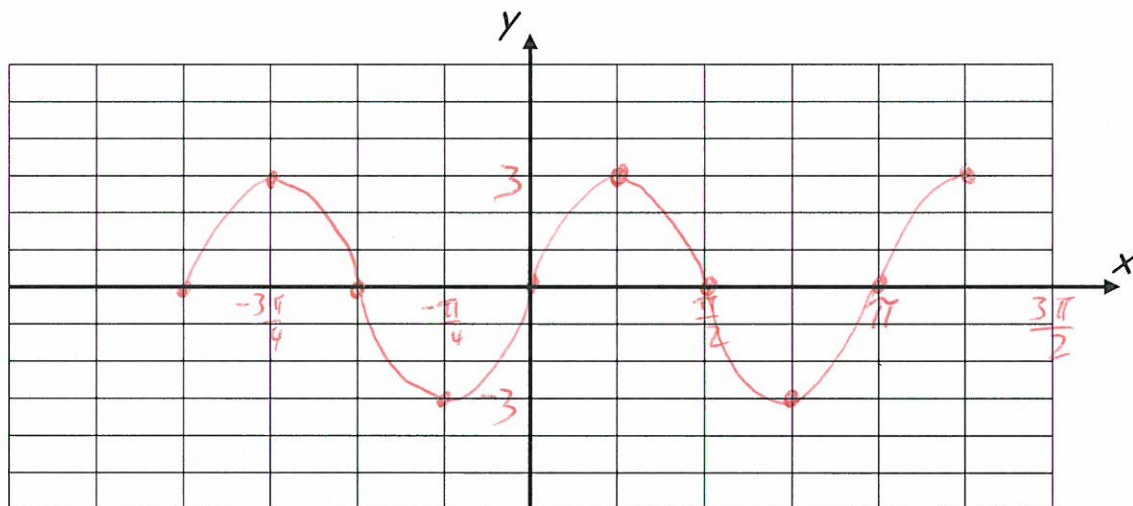


b) $y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$

amplitude= 3

period= π

phase shift= $\frac{\pi}{4}$



7. 10pts

a) What is $\frac{\pi}{5}$ in degrees?

$\frac{\pi}{5} \cdot \frac{180^\circ}{\pi} = 36^\circ$

$\frac{\pi}{5} = 36^\circ$

b) Write 75° in radians as a fraction multiple of π .

$75^\circ \cdot \frac{\pi}{180^\circ} = \frac{15\pi}{36} = \frac{5\pi}{12}$

$75^\circ = \frac{5\pi}{12}$